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ABSTRACT

This document is one part of a critique series that deals with the development and evaluation of course goals in six subject matter areas for grades K-12. The series provides an initial pool of course-level goals that are expected to be of considerable value in assisting educators with goal definition related to curriculum planning and development, instruction, evaluation, and accountability. Goals for the mathematics curriculum are organized according to a subject matter taxonomy. Number systems goals are divided into goals for whole numbers, integers, rational numbers, real numbers, complex numbers, matrices and determinants, vectors, and algebraic expressions. Goals for numeration, mathematical sentences and their solutions, relations and functions, geometry, measurement, sets, logic, probability and statistics, history of mathematics, and use of computational devices are also presented. Four sets of indexes offer the possibility of retrieving course goals by subject matter, knowledge and process, subject area, and career education. Related documents are EA 004 941-2, EA 004 943-948, and ED 061 043. (Author/DN)

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COURSE GOALS IN MATHEMATICS
GRADES K-12

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COURSE GOALS DEVELOPMENT

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MATHEMATICS
TRI-COUNTY GOAL DEVELOPMENT PROJECT

Course Goal Developers

Betty Bialostosky, Reynolds
Robert Buckley, Estacada UH6
Brent Caldwell, Forest Grove
David Clark, Sherwood
Leta Davis, Portland
Allen Gray, Portland
Jan Jacobsen, Portland
Paul Peck, Tigard
Richard Platt, Beaverton
Fred Powell, David Douglas
Katherine Stevens, North Clackamas
Coy Zimmerman, Vancouver

Project Directors

Jack Allen, Multnomah IED
Peter Wolmut, Multnomah IED

Project Coordinators

Robert Allenbrand, Multnomah IED
Walter Hathaway, Portland

Research, Supervision, and Training

Victor Doherty, Portland
Walter Hathaway, Portland

Career Education Resource

Clifford Ferry, Portland
Vivian Grubb, Portland
John Havery, Portland
Karl Hendrickson, Portland
Catherine Williams, Portland

Media Resource

Harold Arendt, Portland
Alice Young, Lynch

Curriculum Resource

William Matson, Portland
Don Rasmussen, Oregon Board of Education

Editing

Clifford Ferry, Portland
Sandra Scofield, NWREL

Project Steering Committee

Jack Allen, Multnomah IED (Curriculum)
George Anderson, Washington IED (Evaluation)
Maynard Christensen, Washington IED (Curriculum)
Victor Doherty, Portland (Evaluation)
Jim Gunderson, Multnomah IED (Data Processing)
Richard Harris, Clackamas IED (Curriculum)
Richard Hermanson, Portland (Data Processing)
Robert Rintoul, Clackamas IED (Data Processing)
Anna Thomas, Clackamas IED (Evaluation)
Clifford Williams, Portland (Curriculum)
Peter Wolmut, Multnomah IED (Evaluation)

INTRODUCTION

A detailed description of the needs to which this collection of resources is responding, the background, the goal types, the goal codes, and the potential uses of this collection of course goals and their supporting materials are to be found in the accompanying booklet, Course Goals General Introduction. The aim of that booklet is to provide users of the course goal collections in Art, Biological and Physical Science, Health Education, Language Arts, Mathematics, Music, Social Science, and Physical Education with a comprehensive guide to the use, revision, and further development of these planning and evaluation resources.

This brief additional introduction has the more practical goals of: (a) presenting a brief orienting overview of the purposes, nature, and potential uses of the products of the Goal Development Project, and (b) demonstrating how to read and interpret the materials in this collection.

Following is a guide to the contents of the introduction:

Page

The purposes, nature, and potential uses of this Course Goal collection:

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The purposes, nature, and potential uses of this Course Goal collection.

Why do we need to state learning goals?

If the basic purpose of education is to help learners to grow and change, then educators and learners must decide and communicate to one another what directions that growth should take and what changes are possible and desirable. Parents, school boards, and the community also have a role to play in influencing educational growth and change. This responsibility can be exercised most rationally if the proposed directions of that growth and change are shared with them in clear and explicit goal statements.

This collection of program and course goals in mathematics is a non-prescriptive resource for educators and boards of education who wish to design and execute learning plans and policies more efficiently and effectively. It is envisioned that school systems will select from this collection those program goals they subscribe to; that schools within a system will select those course goals they believe appropriate to the needs of their communities and students; and that teachers will devise experiences and testing methodologies to meet these goals that are appropriate to the interests and abilities of their students.

What kind of goals are in this collection?

Two types of learning outcomes are included in this collection -- program goals and course goals. They differ in level of generality, with program goals describing broader outcomes and course goals the more specific outcomes relating to them. Also, they differ in the type of planning for which they are suited. Program goals usually serve as guides to planning and organizing programs at district and area levels. Course goals usually serve as guides to planning courses in schools, departments, and classrooms.

At the classroom teacher level the course goals must undergo a final translation into instructional goals and learning experiences. Relying upon the professionalism of teachers, the Goal Development Project has chosen not to intrude into this level, which is concerned with the professional assembling and adapting of resources and methodology needed to achieve the course goals.

How do program and course goals differ from behavioral and performance objectives?

Figure 1 is an illustration from Language Arts of four levels of goals. Examples of behavioral and performance objectives have also been added to show how they differ from the program and course goals defined in the Project.

Note that program and course goals clearly specify a desired learning outcome. But the "behavioral objective" specifies the method of measurement as well as the desired behavior, while "performance objective" adds prerequisites and proficiency level.

The Tri-county Goal Development Project has chosen to produce program and course goals which are suitable for instructional planning, without being constrained by the measurement demands of behavioral objectives or the prerequisites and proficiency levels of performance objectives. Thus, teachers and students are provided explicit statements of possible learnings for which they can accept accountability in ways most suitable to their instructional circumstances. The teacher and student are free to select those methods of achieving selected outcomes which seem most promising within the constraints of their resources and capabilities. This provides for more flexible teaching and learning than teaching machines and other teaching systems based on behavioral and performance objectives. Such an approach places greater demands on the

Figure 1

<table>
<tr><td>System Goal</td><td></td><td>The student is able to communicate with others, both orally and in writing, in a manner that satisfies his need for expression and the requirements of those under whom he may become employed or receive further education.</td></tr>
<tr><td>Program Goal</td><td>P.</td><td>The student is able correctly to apply the conventions of English grammar and usage in speaking and writing.</td></tr>
<tr><td rowspan="2">Course Goal</td><td>K.</td><td>The student knows that special verb forms exist for use with singular and plural subjects.</td></tr>
<tr><td>P.</td><td>The student is able to use appropriate singular and plural verbs with singular and plural subjects.</td></tr>
<tr><td rowspan="2">Instructional Goal</td><td>K.</td><td>The student knows the singular and plural forms of the verb "to be" for present and past tenses:
<table>
<tr><th>S</th><th>P</th><th>S</th><th>P</th></tr>
<tr><td>I am</td><td>We are</td><td>I was</td><td>We were</td></tr>
<tr><td>You are</td><td>You are</td><td>You were</td><td>You were</td></tr>
<tr><td>He, she is</td><td>They are</td><td>He, she was</td><td>They were</td></tr>
</table></td></tr>
<tr><td>P.</td><td>The student is able to use appropriate singular and plural forms (present and past tense) of the verb "to be" with singular and plural subjects in writing sentences.</td></tr>
<tr><td>Behavioral Objective (Method of Measurement Specified)</td><td></td><td>Given 20 sentences, ten with plural subjects and ten with singular subjects, the student will identify the correct number form of the verb (is, are).</td></tr>
<tr><td>Performance Objective (Prerequisites and/or Proficiency Levels Specifie</td><td></td><td>Given 20 sentences, ten with plural subjects and ten with singular subjects, the student will identify the correct number form of the verb (is, are) with at least 90% accuracy.</td></tr>
</table>

ingenuity and professionalism of teachers but has far greater potential because of its consistency with motivational principles and its reliance on the trained judgment of the professional on the scene.

Where did the program and course goals come from?

The program goals were prepared by the Portland School District Evaluation Department in consultation with administrators, teachers, and curriculum specialists throughout the metropolitan Portland area. The course goals were developed by outstanding teachers guided by such models and guides as were available, and supported by tri-county and Oregon State Board of Education curriculum and evaluation personnel.

More than 40 local school districts in the tri-county area of metropolitan Portland are active in the Project. This has been achieved through the leadership of the intermediate educational districts of the three counties. Representatives from Oregon school districts outside the metropolitan Portland area, from Washington State, and from private school systems have also made valuable contributions.

Its broad base of participation strengthens the Project in a number of ways. First, it provides greater financial and personnel support than any single participant could provide. Second, it makes it possible to draw upon a large and nationally representative pool of teacher talent in organizing goal development committees. Third, it provides a widely representative testing ground for the theories and products of the Project. In less than two years there have already been substantial payoffs. Reports indicate that even the critique collections have been used extensively in curriculum development and evaluation the past year and summer.

School districts contribute services of teachers to the Project, using local curriculum funds. Other current sources of support are the Oregon Board of Education, the Small Grants Program of the Regional

Office of the U.S. Office of Education, and curriculum and evaluation funds of the Multnomah, Clackamas, and Washington County Intermediate Education Districts and the Portland Public Schools.

The contributions of the Oregon State Department of Education and USOE are small in terms of the total budget of the Project, but the involvement is significant. The Project's goals are consistent with the State's interests in better educational management practices and instructional improvement. State involvement has already proved important in disseminating products, and the Project has had an influence on state developments in PPBS and educational goals at the legislative and state board level. Finally, USOE involvement provides future potentials for national dissemination and involvement.

What can the goals in this collection be used for?

School systems may use the collections as a yardstick by which to measure the adequacy of goals and objectives already in use. Goals and objectives of local courses of study and textbooks can be contrasted with the goals in this collection to see how complete they are and how well they provide for different interests, abilities, and levels of achievement. They can also be evaluated for conciseness, clarity, and accuracy using these course goals as models.

These kinds of studies can be undertaken by teachers from all levels of a school system (to assure articulation and philosophic unity); across grade levels, divisions, or high school departments (to assure agreement as to goals and ways and means of attaining them) or by individual teachers.

A related use of the goals is as a starting point for reviewing what the schools should teach and the materials to be used to support teaching. The logical sequence of discussions about what is important to teach and learn is to move from broad policy goals to program goals to course goals, with appropriate community-board-staff-teacher-student representations at each level. The taxonomic classifications of this collection can serve as a check on higher order goal formulations, and the goals themselves can function as generators of lower order objectives and instructional plans.

The project provides an important resource for improving the quality and extent of participation of students, parents, teachers, school boards, and other citizens in deciding the mission of the schools. An intensive look at the roles of each participating group in generating, reviewing, contributing to, and approving goals will be a future task of the Project.

Another use of the collection is to provide a basis for teaching-learning accountability. If a school approves all or part of the course goals for its students, grade level, divisional, or departmental representatives may choose from them those that are best suited to individual or group aptitudes and interests.

It is possible for teachers to review goals with each student and contract for their attainment if a completely individualized program is desired. Or, it is possible to stake out a set of goals for target groups (regular classes, special classes, mini-courses, etc.). In any event, the goals themselves are sufficiently explicit that means of teaching them and of evaluating their attainment can be devised and applied individually or to groups to suit the needs of teachers and management.

Another use of the collection is the rewriting and development of courses and curricula. By making curricular options explicit and sharable, the collection can help in the development of new or modified courses of instruction and the design or redesign of curricular experiences. One important example of curriculum development fostered by this collection is cross-disciplinary education. Probably no concept is currently more abused than "interdisciplinary education." While the goals of subject matter learning are at least implicit in the textbooks and other materials used by teachers, the goals of interdisciplinary education do not have even that questionable point of tangible reference. The Tri-county Project, through its extensive coding and retrieval systems, permits selection of goals in terms of various combinations of subject matter, educational level, types of knowledge and process, career education program goals, concepts and values, and index words. This system provides important cues for interdisciplinary planning. The goals, although printed in subject collections such as science, social science, mathematics, music, etc., may be related and grouped in and across subjects through computer retrieval by requesting those goals bearing one or more of the seven code parameters. Thus, for example, a teacher interested in a unit on marine biology can request goals dealing with related concepts in science, social science, language, mathematics, or any other subject field.

A final use of this collection is for long-range planning and systematic control of educational development. The past few years in education have demonstrated that few results of experimentation and development are transportable. The inability of educators to define clear, unambiguous statements of desired learning outcomes is an important underlying cause. The Tri-county Project is establishing sets of goals that may be used consistently for instructional planning and evaluation. The sets are open and are added to each time teachers or curriculum planners specify appropriate learnings not represented in the original collections. However, any statement admitted to the collection undergoes a rigorous process of statement, definition, and coding to insure that its utility to teachers is equal to goals already in the collection.

These collections will support all curriculum development activities in the Portland School System within a year or two, and in many other school districts in the tri-county area as well. The stability this will provide educational experimentation and development is apparent. The power of the goal collections themselves in promoting good educational planning and the ease and convenience it affords teachers in that planning is equally evident.

Other uses can be cited, but districts will discover these. In all of the above activities, districts are invited and encouraged to use the collection selectively and to add their own goals wherever this collection is insufficient to their needs. We hope that where they do add and modify, they will use the feedback forms and contribute to the expansion and improvement of the original collection.

Will help be available for evaluating the attainment of the goals in this collection?

The principal measurement-related product sought by the project developers is a set of test items related to each course goal. This set is to be so comprehensive that any teacher who selects a course goal and translates it into one or more instructional goals will be able to retrieve items, or at least examples of items, appropriate to assess the attainment of his instructional goals.

The Project is beginning to define evaluation models appropriate for assessment of goals in each of the classes of knowledge and process. These models will be used to guide both psychometricians and teachers in the development of criterion referenced test items appropriate for measuring each type of knowledge and process. Teachers using the course goals during the period the items are being developed will be asked to supply copies of their periodic and final examinations to provide materials for a comprehensive set of test items. Teacher aids for test item development based on the different goal types are being prepared to insure the quality of the item bank. As soon as theoretical formulations relating to values, generalizations, and concepts are refined and consistent, similar work will begin in developing evaluation models and items for those classes of learning. This work should take two to five years to complete, depending on resources.

Is this the final version of the program and course goals?

No. In the development of both the program and the course goals, an effort was made to make them comprehensive, realistic, and immediately applicable to schools as they are currently organized. At the same time, these goals and the taxonomy are to be revised and improved as they are subjected to use and scrutiny by teachers and curriculum personnel. This is to be accomplished through the feedback instrument distributed with these goals.

As time passes, new goals will be called for. For this reason a provision is being made for the continual review and revision of the goals. This will be largely dependent on feedback from the field. Thus, what is being created is a complete, dynamic, open system for goal-based learning and evaluation. Such a system will be a useful resource to all those seeking to improve their understanding of what should be learned, how it should be learned, and how evaluated.

How to read and interpret the materials in this goal collection.

Following this introduction there are four sets of indexes for retrieving course goals (indicated by four different colors): subject matter taxonomy, pink; knowledge and process classifications, yellow; subject area program goals, blue; and career education program goals, green.

Codes on the course goals refer to the materials on the colored pages. The colors are to help you find the meaning of a code found beside a course goal. Below is a description of how to read and interpret a page of course goals and its codes.

The bulk of the pages in this collection are taken up with the course goals themselves and their codes. Since our aim here is to learn how to read, interpret, and use these goals, let us look at and discuss a sample page of them taken from the music collection. The format of the page is the same for the mathematics collection. (Please see following page.)

The number headings of the left hand column (4. Listening to Music and 4.2 Responses) are those sections and subsections of the subject matter taxonomy under which the goals on this page are classified. The subject matter taxonomy which is to be found on the pink pages in the front of the book serves as a table of contents for this collection.

By looking through the taxonomy, a user can find what topics are covered and can turn to those in which he is interested. Also, the headings may be used along with one or more of the other codes to retrieve subsets of goals from the computerized storage system. Finally, the taxonomies form a comprehensive but brief overview of the topics in each subject area judged important in K-12 curricula. As such, they form a valuable and convenient tool for curriculum and materials review and planning.

The next thing we note in column (1) under the heading "4.2 Responses (cont.)" are the Course Goals themselves. Some goals in this column have a

MUSIC

(1)

4. Listening to Music

COURSE GOALS	(2) Level P/I/U/H	(3) Knowledge or Process Classifications	(4) Subject Area Program Goals	(5) Career Education Program Goals	(6) Other Related Content Taxonomy Headings	(7) (C) Concept/ (V1, V2) Value Words
4.2 Responses (cont.)						
The student knows that the ability to listen effectively can be acquired through attention to specifics and through experiences.	I U H	G2 K8	1a 4a 4b		4.23	(C) Intellect (V1) Aesthetic perception
The student knows reasons why musical appreciation requires an auditory impression to be retained in the memory (i.e., the composition cannot be seen in its entirety, as is possible in the visual arts).	I U H	K7 K8	1a 4a 4b	4a 4d	4.23	(C) Aural memor (V1) Aesthetic perception
The student knows that auditory impression is retained in the memory through attention, repetition, and familiarity.	I U H	K3 K8	1a 4a 4b		4.23	(C) Aural memor
The student knows that the greater his knowledge of music, the more he can listen for and the greater his potential for enjoyment.	U H	C2 K3 K8	1a 4a 4b	3c	4.23	(C) Mood (C) Elements, music (V1) Pleasure (V1) Self-knowledge
The student is able to direct his attention toward specific factors in the music (e.g., mood, style, subject matter, tonality, instrumentation).	U H	P33 P62 P65	1a 4a 4b	5a		(C) Elements, music (V1) Aesthetic perception
The student is able to distinguish specific factors in music he hears (e.g., characteristics of melody, structural organization, expressive characteristics).	P I U H	P33 P45	1a 4a 4b	5a		(C) Elements, music (V1) Aesthetic perception
The student knows that his knowledge of the conventions of music establishes expectancies in the listening process.	I U H	K2 K8	1a 3a 4a 4b		8.12	(C) Elements, music (V1) Aesthetic perception (V1) Self-knowledge

bracket to their left. The goals inside the bracket are logically related and may be viewed as a unit.

The column (2) on the page as we move from left to right is headed "Level P/I/U/H" (primary, intermediate, upper, and higher). This code provides the teacher or curriculum planner an estimate of the level or levels at which the learning is appropriate. Many times the nature of the goal suggests continued learning over several levels, in which case all levels involved are coded. These indications of level are suggestive only, for it is evident that the appropriate time for learning varies with the interests and abilities of students.

The third column (3) is headed "Knowledge or Process Classifications." The classifications referred to are described at the front of the book on the yellow pages. All goals are roughly classified as knowledge or process depending upon whether they deal with something that is to be known or something the student is able to do. All goals, therefore, begin with the words, "The student knows..." or "The student is able to..."

The familiar knowledge/process distinction is further subdivided into twelve knowledge and seventy-nine process categories to which all course goals have been coded. It will be noted that these classifications owe a partial debt to earlier researchers; notably, Benjamin Bloom, David Krathwohl, Robert Glaser, Henry Walbesser, and Ralph Tyler in Education; Robert Gagné and Robert Miller in Psychology; Jean Piaget and Jerome Bruner in Child Development; and others.

At this point the reader may question the reason for the rather detailed and elaborate system of classifying educational outcomes that has evolved during the Project. We have found that providing teachers with these classification systems has resulted in a more critical approach to the writing of goals. A teacher in attempting to place a goal in its appropriate category may find that its intent is clearly related to one of the categories but its form of expression does not immediately identify it with that category. By rewording the goal, the teacher brings the true intent of the goal into sharper focus, and in almost every instance improves its meaning and clarity. We have also found that the detailed classifying of knowledge and process goals provides insight into alternative ways of using them for teaching and evaluation. For example, the G2 and K8 beside the first goal on the preceding page indicate that it may be taught and evaluated either as a simple generalization (G2) or as a goal about the causes of effective listening (K8). Finally, work has already begun in analyzing and suggesting to teachers the types of measurement appropriate for each type of knowledge goal. This work will be extended to process learning as rapidly as resources permit.

Column (4) on the page is headed "Subject Area Program Goals." In this column we find the number of one or more of the program goals found in the front of this book on the blue pages. The definition of this type of goal and its relation to course goals was discussed earlier. Here it is enough to recall that program goals are more general than course goals and that a set of program goals should constitute a description of the major overall learning outcomes expected from a program. Each course goal is cross coded to the program goal(s) to whose attainment it is most directly related.

Column (5) on the page is headed "Career Education Program Goals." In this column we may find the code of one or more of the career education program goals found in the front of the book on the green pages. Career education, as envisioned by the coders, concerns the total life of an individual, including day-to-day living, vocation, avocation, and leisure. Nearly every course goal bears at least an indirect relationship to career education viewed in that manner. Only those course goals, however, which have a "direct" relationship to a career education program goal have been coded to that program goal.

A "direct" relationship was interpreted to exist between a course goal and a career education program goal if a teacher could easily and naturally attach some career meaning to the instructions relating to that course goal and thus readily integrate the teaching of career education into teaching his subject. The restriction of the codings to direct relationships as just defined means that codings to career education program goals are relatively rare in the goals written under the more detailed and technical parts of a subject's taxonomy. In these relatively specific and technical areas, Career Education Program Goal 5a, which deals with the basic skills necessary for job entry and is found only in process goals, is often the only code found in this fifth column. This is even more true in the mathematics collection than in the collections in some of the other subject matter areas. Since mathematics is basic to so many aspects of career education, it is especially difficult to single out special goals as being more directly related than others to program goals of career education. Therefore, except for a few goals which deal explicitly with careers in the latter sections of goals, 5a is used almost exclusively in this critique's collection to cross code to career education. The validity and possible extensions of this approach to integrating mathematics and career education will continue to be explored by the Project.

A reader should not assume that because a course goal is cross coded to a career education program goal that he should make an effort to relate it to that aspect of career education in every case. That is up to himself and the policies to which he is responsible. This coding provides suggestions, not prescriptions for curriculum planning and teaching.

A teacher may use this coding as a help to integrating a discipline and career education and vice versa, by asking himself the following question: "When I am teaching this goal, is there some aspect of career education that can be usefully and naturally brought to the attention of my students?" The cross coding, where it appears, suggests there may be and what the aspect is.

The career education code used with these goal collections makes them the first operational resource for "integrating career education and the rest of the curricula." Naturally a great deal of work has to be done to refine and extend the beginning which the present cross codings represent.

The coding "Other Related Content Taxonomy Headings" under column (6) is provided since goals are often rightly classified under more than one subject heading. The numbers in this column refer to the taxonomy on the pink pages at the front of the book. For purposes of computer retrieval, it is possible to request all goals which deal with a particular subject heading, and to extract not only the goals placed under that heading, but also all other goals cross-referenced to it wherever they are located in the collection. While this capability presently exists only within a subject field, it later will be provided among subject fields.

Column (7) on this page is headed "(C) Concept/(V1, V2) Value Words." This form of code is one of the newest and potentially most useful ways to describe and retrieve sets of goals, especially for interdisciplinary learning. Although explicitly singling out the concepts and values dealt with in goals is theoretically very interesting and useful, in practice it is very difficult since no valid lists of such concepts and values exist in the various subject areas. Accordingly, the codings applied in this critique edition should be viewed as experimental attempts made to solicit constructive criticism. Those who feel that a goal deals with a concept or value according to the definitions and procedures described below and do not find it coded are invited to so inform the revision committees through the feedback forms provided with this collection.

Words chosen to characterize values and concepts represent residuals of experience that influence the way individuals perceive and behave. Thus, the word freedom connotes certain behaviors associated with the ideal state. Likewise, a word like honesty characterizes a set of behaviors which viewed from a societal perspective characterizes an individual as honest. From an educator's point of view, the only resources available to help students acquire the desired concepts and behavioral tendencies are the knowledge and process learnings planned for and with students.

The word designating the major concepts to which a goal relates were written beside that goal in the seventh column. Words identifying concepts are preceded by "(C)" to distinguish them from the value words found in the same column.

A glance through the subject taxonomy on the pink pages at the front of the book reveals many headings which themselves are concept words. These headings have not been repeated as concept words on every goal under that heading, but only on those goals which bear the most direct and general relationship to the concept designated.

Especially important in considering the nature of values is the distinction between the instrumental processes of clarifying and forming values (V2) and values as end products to be inculcated and strived toward (V1). The curricular and methodological implications of teaching toward values as end products are entirely different from those concerned with the processes of value clarification and formation.

In helping students acquire and strive to attain values (V1), the educator must rely upon teaching knowledge and skills that have a logical bearing upon these values. Where he is concerned with the teaching of value clarification and formation processes (V2), he must teach such conventional skills as verifying information, relating information to criteria, and other methods of clarifying personal and social values by which the clarification, interpretation, and internalization of information can be accomplished. These are the same processes found in the Inquiry and Problem Solving Processes Classification on the yellow pages at the first of the book and are coded in column (3).

The type of values coded in column (7) of this goal collection is type (V1). Where a goal may be used to inculcate or help a student attain a value, the value is named in this column and a "(V1)" is written in front of it. Where a process related to value formation is dealt with in a goal, it will be a process goal. The process will be indicated by the process code in column (3).

The nature of mathematics has limited the number of (V1) value codings in this collection. With few exceptions they are to be found in the final four sections of goals (8-11). The values inculcated and strived toward in mathematics such as mathematical elegance, rationality, fidelity to logic, rigor, etc., for the most part underlie the subject matter of mathematics rather than being dealt with directly in its subject matter. The Project will continue to explore the relationship between mathematics and the very important values it supports and ways to make their relationship explicit.

Another useful code is the Index Word. Although it does not appear on the printed page, it is keyed to each goal for retrieval in much the way documents are coded for retrieval in the familiar ERIC retrieval system. Users will have available lists of index words by discipline and across disciplines.

A most important set of materials in this manual is the Feedback Instrument. This instrument calls for the minimum information we need from you, the user, if we are to refine and expand the collection and improve its value to all users. Additional input is welcomed by phone, word of mouth, carrier pigeon, etc., after you have discussed and tried out this resource in your district. Ultimately, the success of the project is dependent on this input.

SUBJECT MATTER TAXONOMY

MATHEMATICS TAXONOMY

- 1.0 Number systems
  - 1.1 Whole numbers
    - 1.11 Addition
    - 1.12 Subtraction
    - 1.13 Multiplication
    - 1.14 Division
    - 1.15 Cardinal numbers
    - 1.16 Ordinal use
    - 1.17 Ordering
  - 1.2 Integers
    - 1.21 Addition
    - 1.22 Subtraction
    - 1.23 Multiplication
    - 1.24 Division
    - 1.25 Absolute Value
    - 1.26 Order
  - 1.3 Rational numbers, positive, zero, and negative
    - 1.31 Addition
    - 1.32 Subtraction
    - 1.33 Multiplication
    - 1.34 Division
    - 1.35 Absolute Value
    - 1.36 Order
    - 1.37 Ratio, proportion, and percent

- 1.4 Real numbers
    - 1.41 Operations
    - 1.42 Absolute Value
    - 1.43 Order
    - 1.44 Limits and sequences
- 1.5 Complex numbers
- 1.6 Matrices and determinants
- 1.7 Vectors
- 1.8 Algebraic Expressions
    - 1.81 Polynomials
    - 1.82 Rational expressions

2.0 Numeration

- 2.1 Base systems
- 2.2 Decimal systems
- 2.3 Other systems
- 2.4 Scientific and exponential notation
- 2.5 Rational number names
- 2.6 Number theory

3.0 Mathematical sentences and their solutions

- 3.1 Equalities
- 3.2 Inequalities
- 3.3 Systems
- 3.4 Word problems and applications

4.0 Relations and functions

- 4.1 Mapping
- 4.2 Inverse relations and functions
- 4.3 Algebraic functions
- 4.4 Exponential functions

- 4.5 Logarithmic functions
- 4.6 Trigonometric (circular) functions
- 4.7 Probability functions
- 4.8 Other functions

5.0 Geometry
- 5.1 Plane and space
- 5.2 Coordinate geometry
- 5.3 Transformational geometry
- 5.4 Other geometries

6.0 Measurement
- 6.1 Systems and categories
- 6.2 Accuracy
- 6.3 Measurement instruments

7.0 Sets
- 7.1 Terminology
- 7.2 Operations on sets

8.0 Logic
- 8.1 Terminology
- 8.2 Statements
- 8.3 Reasoning
- 8.4 Abstract mathematical systems

9.0 Probability and statistics
- 9.1 Sampling and data collection
- 9.2 Measures
- 9.3 Combinatorial
- 9.4 Probability theory
- 9.5 Interpretation

10.0 History of Mathematics
    10.1 Early Period
    10.2 Greco-Roman Period
    10.3 Middle Age Period
    10.4 Modern Period
11.0 Computational Devices
    11.1 Computers
    11.2 Slide Rules
    11.3 Other Devices

KNOWLEDGE
AND
PROCESS
CLASSIFICATIONS

-Knowledge Categories-

G1 Principals and Laws

G2 Simple Generalizations

K1 Conventions: Names and Nomenclature

K2 Conventions: Symbols, Rules, Standardized Processes, Definitions

K3 Properties, Parts, Characteristics, Features, Elements, Dimensions

K4 Trends and Sequences

K5 Similarities and Differences, Discriminations, Classifications

K6 Contexts, Locations, and Orientations

K7 Operations, Methods of Dealing with, Functions

K8 Cause and Effect Relationships (Costs and Benefits)

K9 Criteria or Standards

K10 Non Cause-Effect Relationships

-Inquiry-Problem Solving Processes-

P1 Input Acquiring Information

- P11 Viewing
- P12 Hearing
- P13 Feeling (tactile)
- P14 Smelling
- P15 Tasting
- P16 Using sense extenders

P2 Input Verification Insuring Validity and Adequacy

- P21 Evaluating authoritativeness of sources
- P22 Evaluating logical consistency and accuracy
- P23 Evaluating relevance to desired learning purposes
- P24 Evaluating adequacy for acting or deciding (comprehensiveness and depth)

P3 Preprocessing Organizing Information

- P31 Labeling, naming, numbering, coding
- P32 Recording, listing
- P33 Classifying, categorizing, grouping, selecting, according to criteria
- P34 Ordering, sequencing
- P35 Manipulating, arranging, transforming, computing
- P36 Estimating
- P37 Summarizing, abstracting

P4 Processing I Interpreting Information (drawing meaning from data)

- P41 Decoding verbal and non-verbal symbols (reading and literal translating)
- P42 Inferring, interpolating, extrapolating
- P43 Analyzing
- P44 Associating, relating, equating
- P45 Comparing, contrasting, discriminating
- P46 Synthesizing
- P47 Testing against standards or criteria
- P48 Generalizing

P5 Processing II Using information to Produce New Information

- P51 Theorizing, predicting
- P52 Formulating hypotheses
- P53 Testing hypotheses
- P54 Revising hypotheses

P6 Output I Acting on the Basis of Information

- P61 Reacting
- P62 Making decisions
- P63 Solving problems
- P64 Restructuring values (adapting, modifying)
- P65 Restructuring behavior (adapting, modifying)
- P66 Encoding verbal and non-verbal symbols prior to communication
- P67 Creating on the basis of knowledge and process

P7 Output II Communicating Information

- P71 Vocalizing (non-verbal)
- P72 Gesturing, moving
- P73 Touching
- P74 Speaking
- P75 Writing
- P76 Using art media (painting, drawing, sculpting, constructing, etc.)
- P77 Dramatizing
- P78 Singing, playing instruments
- P79 Dancing

SUBJECT AREA
PROGRAM GOALS

MATHEMATICS PROGRAM GOALS

1. The student knows and is able to use the symbols, elements, operations, and structure of the following number systems: whole numbers, integers, rational numbers, real numbers, complex numbers and other systems, both finite and infinite.
    a. The student is able to use the language and symbolism of sets to express properties of number and number operations, points, objects, and ideas and relationships among them.
    b. The student knows that numbers or algebraic expressions may be written in many forms and is able to rewrite them from one form to another.
    c. The student is able to use the properties of equality and inequality to relate numbers.
    d. The student knows the properties of and relationships among the fundamental operations.
    e. The student is able to compute with accuracy and efficiency in operating with numbers and expressions.
2. The student is able to use the concept of "function" as defined mathematically, in interrelating concepts, topics, and branches of mathematics.
    a. The student is able to use graphs, tables, algebraic or trigonometric sentences to show the relationship between two sets of numbers, or elements other than numbers, and is able to identify relationships that are mathematical functions.
    b. The student is able to use the knowledge of "function" as represented by graphs, tables, and mathematical statements for the solution of real and simulated problems.
    c. The student is able to read and interpret graphs and tables found in magazines, newspapers, bulletins, and reports.
3. The student knows the historical and cultural development of counting, measuring, and of mathematical symbols and systems.
4. The student is able to use mathematical symbols, systems, and operations to the solution of quantitative problems.
    a. The student is able to estimate the solution of a quantitative problem.
    b. The student is able to translate a problem situation into a mathematical sentence or model, find a solution for the model, and reinterpret the mathematical solution in the context of this problem situation.

5. The student knows and is able to apply the properties of geometric figures.

    a. The student is able to associate a number called length with a line segment, a number called area with a closed surface, and a number called volume with a closed surface and its interior.

    b. The student is able to visualize or approximate distances and locations in space.

    c. The student is able to describe point sets algebraically and algebraic sentences geometrically using a coordinate space.

    d. The student is able to use inductive and deductive reasoning to derive geometric properties.

    e. The student is able to use properties of geometric figures in solving real and simulated problems.

6. The student is able to use logic symbols and operations in solving problems involving logical inferences.

    a. The student is able to synthesize rules of logic to prove or disprove statements.

    b. The student is able to recognize instances of the use of faulty logic.

    c. The student is able to make valid generalizations from specific instances.

    d. The student is able to organize logical arguments.

7. The student knows and is able to use the mathematics of probability and statistics.

    a. The student is able to collect and tabulate data, compute statistical indices descriptive of the data, and interpret the data gathered.

    b. The student is able to decide which statistical indices and graphical representations are most suitable for a given situation.

    c. The student is able to make probabilistic inferences from statistics.

    d. The student is able to recognize misuse of statistics.

8. The student is able to select and use support technology such as calculators, computers, and slide rules in the solution of mathematical problems and problems which require mathematical solutions.

9. The student is able to measure things which are quantitatively describable.

    a. The student is able to use conventional units of measurement and select the unit of measurement which is most appropriate and efficient for a particular purpose.

    b. The student is able to communicate using measurement ideas and terminology.

CAREER EDUCATION
PROGRAM GOALS

CAREER EDUCATION PROGRAM GOALS

	Awareness K-6	Exploration 7-10	Preparation 11-12
1. Attitudes and Values Toward Self and Others	X	X	X
2. Attitudes and Values Toward Work	X	X	X
3. Career Education and the Total Curriculum	X	X	X
4. Career Exploration		X	X
5. Career Preparation		X	X
6. Career Placement and Employment			X

Regardless of the instructional level at which each group of program goals is introduced, continuous development and reinforcement through the remaining years of education is expected.

CAREER EDUCATION PROGRAM GOALS

1. Attitudes and Values Toward Self and Others

    a. The student knows the physical and emotional benefits of understanding and respecting self and others throughout life.

    b. The student knows that the major sources of understanding, acceptance, and respect of self are understanding, acceptance, and respect for others.

    c. The student knows that success in his career is dependent on satisfactory interpersonal relationships with employers and fellow workers.

2. Attitudes and Values Toward Work

    a. The student knows the personal, social, economic, and political reasons for work in our society.

    b. The student knows that work is a dignified human activity which gives rights to and requires responsibilities from its participants.

    c. The student knows that in our society he is dependent on the goods and services of others for his welfare and survival.

3. Career Education and the Total Curriculum

    a. The student knows that skill in job exploration, selection, and preparation can lead to continuing career enhancement and personal fulfillment.

    b. The student is able to identify career alternatives, select those consistent with his values and goals, and implement chosen courses of action.

    c. The student knows the physical and psychological reasons for seeking a balance between work and leisure activities.

4. Career Exploration

    a. The student is able to evaluate his aptitudes, interests, and abilities in exploring career opportunities.

    b. The student knows the major factors that may affect his career opportunities and decisions (e.g., physical, social, economic, educational, cultural, and technological).

    c. The student knows that individuals can learn to function effectively in a variety of occupations.

d. The student knows that every career has entry, performance, physical, attitudinal, and educational requirements.

e. The student knows that career choice may help determine friends, associates, and status in the community.

f. The student is able to select a tentative career choice based upon exploration of a wide variety of occupations.

g. The student knows that career choice affects the amount and type of leisure activity that may be pursued.

5. Career Preparation

a. The student is able to develop and apply the basic skills and behaviors required to perform one or more entry level jobs.

b. The student is able to employ the following organizational skills appropriate to the career of his choice:

1. identify the objectives of a task
2. specify the resources required
3. outline the steps necessary for completion
4. perform the actual operations
5. evaluate the final product

6. Career Placement and Employment

a. The student is able to make an assessment of the labor market to determine opportunities that will advance his career.

b. The student knows the educational opportunities that exist beyond grade 12 for the enhancement of his career skills and his personal development.

c. The student knows the advantages and responsibilities associated with working independently, as a member of a team, and under direct supervision.

d. The student knows that the acceptance of a task requires the acceptance of responsibilities to himself and others.

e. The student knows the opportunities for vertical and lateral mobility within his career cluster.

COURSE GOALS

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge of Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Courses Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1. Number Systems						
The student knows the location and use of print and nonprint materials related to number systems in mathematics (e.g., card catalog: "Numbers, Theory of", "Arithmetic", "Algebra"; Reader's Guide: "Arithmetic", "Numbers, Theory of", "Mathematics"; Area and Building Audio-Visual Catalogs: "Mathematics, Rational Number", "Mathematics, Integer", "Mathematics, Arithmetic"; Periodical: Arithmetic Teacher).	P I U H	K6	1a 1b 1c 1d 1e 1f 1g 1h			(C) Resources, mathematics (VI) Inquiry
The student knows that the field of mathematics is considered a universal language.	I U H	G2 K2 K3	1a			(C) Language, universal
The student knows that a mathematical system consists of a set of elements and one or more operations on the elements.	H	K2 K3	1a 1d			(C) Structure, math
The student knows that a binary operation on a set is a rule whereby to each ordered pair of elements in the set there corresponds exactly one element.	P I U H	K2 K7	1d			
The student knows that the operations of addition, subtraction, multiplication and division are binary operations.	P I U H	K3 K7	1d			(C) Operations, math
The student knows the meaning of the symbols for mathematical operations (including +, -, x, ÷, ∩, ∪, √).	P I U H	K2 K7	1a			(C) Operations, math
The student knows the order in which operations are performed.	P I U H	K2 K4 K7	1d 1e			(C) Operations, math
The student knows symbols for grouping numbers such as braces, brackets, and parentheses (Symbols of Inclusion).	P I U H	K2	1a			

I. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge of Process Classification	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Heading	(C) Concept/(V1, V2) Value Word
I. Number Systems (Cont.)						
The student knows the meaning of the terms associated with computation (including sum, total, addend, difference, factor, product, dividend, division, quotient).	P I U H	K1 K2	1a			
The student knows a well defined operation in a set is one that produces a unique element within the set.	I U H	K2 K7 K8	1d			
The student knows that a closed operation in a set is one that produces an element in the set (Property of Closure).	P I U H	K2 K3 K7 K8	1d			
The student knows the commutative property for an operation in a set, that is, for elements a and b of a set, and an operation *, a * b = b * a .	P I U H	K2 K3 K7 K8	1d			
The student knows the associative property for an operation on a set, that is, for elements a, b and c of a set, and an operation *, a * (b * c) = (a * b) * c.	P I U H	K2 K3 K7 K8	1d			
The student knows the distributive property for operations on a set, that is, for elements a, b and c of a set, and the operations # and *, a * (b # c) = (a * b) # (a * c).	P I U H	K2 K3 K7 K8	1d			
The student knows that if a set contains an identity element, then an operation with any given element and the identity element produces the given element.	I U H	K2 K3 K7 K8	1a 1d			
The student knows that if a set contains an inverse element, then an operation with an element of that set and its inverse produces the identity element for the set.	I U H	K2 K3 K7 K8	1a 1d			

I. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/(V1, V2) Value Words
I. Number Systems (Cont.)						
The student is able to apply the number properties as an aid to simplify computation (including grouping of tens in addition, quick recognition of zero factors or dividends).	P I U H	P35 P41 P65	1b 1e			
The student knows the properties of major mathematical systems including group, integral domain, ring, and field.	U H	K3	1a 1d			(C)Structure, math
The student is able to determine whether a system satisfies the properties for a particular mathematical structure (including group, ring, and field).	H	P33 P45 P47	1a 1d			(C)Structure, math
The student knows the definition of the reflexive, symmetric, and transitive properties.	I U H	K2 K3	1c 2a		5.0	(C)Equivalence
The student knows that for two elements a and b of a set of numbers, one and only one of the following is true: $a = b$, $a < b$, or $a > b$. (Trichotomy Law)	I U H	G1 K1 K2	1a 1c			
The student knows the number line is a symbolic representation of the set of real numbers.	P I U H	K2 K7	1a 2a			(C)Number, relationships
The student knows that a set is dense if for any two elements a and b, there is an element between a and b.	U H	K2 K3 K9	1a			
The student knows that cardinal number refers to the number of elements of a set whereas ordinal number refers to position in an ordered set.	P I	K2 K5	1a			(C)Cardinality (C)Ordinality

MATHEMATICS

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.1 Whole Numbers						
The student knows the set of whole numbers is an infinite set that can be represented by $\{0, 1, 2, 3 \ldots\}$.	P I U H	K2	1a			(C) Infinity, math
The student knows the set of natural (counting) numbers $\{1, 2, 3, \ldots\}$ is a subset of the set of whole numbers $\{0, 1, 2, 3, \ldots\}$.	P I U H	K2 K5	1a			(C) Structure, math
The student knows the set of whole numbers is not dense since there is no whole number between two consecutive whole numbers.	I U H	K3 K9	1a			
The student knows that there are many names for the same whole number (e.g., 26, 20 + 6, twenty-six, XXVI).	P I U H	K2	1a 1b		2.0	(C) Numeration
The student knows each element of the set of whole numbers has a successor, i.e., $\{0, 1, 2, 3, \ldots\} = \{0, 0 + 1, 1 + 1, 2 + 1, \ldots\}$.	P I U H	K3 K4	1a			(C) Number relationships (C) Numeration
The student knows that the set of whole numbers may be represented on a number line.	P I U H	K3 K5	1a 2a			(C) Number relationships
The student is able to locate the set of whole numbers on the number line.	P I U H	P31 P44	1a 2a			(C) Number relationships
The student is able to represent mathematical operations with whole numbers on a number line.	P I U H	P31 P44 P66	1a 2a			(C) Operations, math

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.1 Whole Numbers (Cont.)						
The student knows that the identity elements for the set of whole numbers are 1 for multiplication and 0 for addition.	P I U H	K2	1a 1e			(C)Number, relationship
The student knows addition-subtraction and multiplication-division are inverse operations in the set of whole numbers.	P I U H	K3	1d			(C)Operations, math
The student knows that in the set of whole numbers the four fundamental operations are well-defined since a unique result of the operation is in the set.	U H	K3	1a 1d			(C)Operations, math
The student knows the four fundamental operations may be represented by combining or partitioning sets.	P I U H	K2 K5 K7	1d			(C)Operations, math
The student is able to use properties of number, numeration, and operations to derive some computational algorithm of his own.	I U H	P67	1e		2.0	(C) Operations, math
The student is able to estimate whether a sum, difference, product, or quotient is reasonable.	P I U H	P36	1e			(C) Operations, math

MATHEMATICS

1. Number Systems

1.1 Whole Numbers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Headings	Taxonomy (C) Concept/ (V1, V2) Value Words
1.11 Addition						
The student knows the additive combinations of the whole numbers "0" through "9".	P I U	K2 K3 K7	1b 1e			
The student knows ways in which addition is defined (including counting, combining of sets, displacement on a number line).	P I U H	K2	1a 1d			
The student knows the set of whole numbers with respect to addition is closed, commutative, associative and that multiplication is distributive over addition.	P I U H	K3	1b 1d 1e			(C) Structure, math
The student is able to regroup (carrying) in addition.	P I U H	P33 P35	1b 1e		2.0	
The student knows that only places of like value may be added.	P I U H	K2 K5 K7	1b 1e		2.0	
The student is able to add whole numbers	P I U H	P35	1b 1e	5a		

1. Number Systems
1.1 Whole Numbers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/(V1, V2) Value Words
1.12 Subtraction						
The student knows the subtraction combinations of the whole numbers 1 through 9.	P I U H	K2 K3 K7	1b 1e			
The student knows ways in which subtraction is defined (including as the inverse of addition, finding a missing addend, partitioning of sets).	P I	K2	1d			
The student knows that in the set of whole numbers the properties of closure, commutativity and associativity are not valid for subtraction.	P I U H	K3	1d 1e			(C) Structure, math
The student is able to regroup (borrowing) in subtraction.	P I U H	P33 P35	1b 1e			
The student is able to use rules of place value and regrouping to subtract whole numbers.	P I U H	P35	1b 1e			
The student is able to check subtraction of whole numbers by addition.	P I U H	P24 P35 P47	1e			
The student is able to subtract whole numbers.	P I U H	P35	1b 1e	5a		

1. Number Systems
1.1 Whole Numbers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.13 Multiplication						
The student knows the basic rules of multiplication in the set of whole numbers (including multiplying by 10, 100, 1000).	P I U H	K2 K3 K7	1e			
The student knows that in the set of whole numbers multiplication may be defined as repeated addition (4 x 2 = 4 + 4 = 2 + 2 + 2 + 2).	I U H	K2 K7	1d 1e			
The student knows that in the set of whole numbers multiplication is closed, commutative, associative, and distributive over addition.	P I U H	K3	1a 1d 1e			(C)Structure, math
The student knows that the product of two whole numbers, a and b, is zero, if and only if, a or b is zero.	P I U H	K3 K7 K9	1e			
The student knows an algorithm for multiplication in the set of whole numbers.	P I U H	K2 K7	1d 1e			
The student is able to multiply whole numbers.	P I U H	P35	1e	5a		
The student is able to check the accuracy of a product (including the use of the commutative property, distributive property, inverse operations, repeated addition).	I U H	P24 P35 P47	1d 1e			

1. Number Systems

1.1 Whole Numbers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.14 Division						
The student knows the inverse relations of the multiplication tables.	P I U H	K2 K3 K7	1a 1e			
A student knows that division of whole numbers may be defined as repeated subtraction or as the inverse of multiplication.	P I U H	K2 K7	1d 1e			
The student knows that for whole numbers a, b, and c, $a \div b = c$ if and only if $c \cdot b = a$ and $b \neq 0$.	U H	K2 K7 K9	1b 1d			
The student knows that in the set of whole numbers, division by zero is undefined.	P I U H	K3	1a 1d		1.0	
The student knows that in the set of whole numbers the properties of closure, commutativity and associativity are not valid for division.	I U H	K3	1d 1e			(C)Structure, math
The student knows an algorithm for division of whole numbers.	P I U H	K2 K7	1b 1e			
The student is able to divide whole numbers.	I U H	P35	1b 1e	5a		
The student is able to check the accuracy of a quotient (using inverse operations, repeated subtraction, applying the distributive property).	I U H	P24 P35 P47	1d 1e			

1. Number Systems
1.1 Whole Numbers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.15 Cardinal Numbers						
The student knows that a cardinal number tells how many objects a set contains.	P I U H	K2	1a		7.0	(C)Cardinality
The student knows that there may be several ways to count the members of a set but there is only one cardinal number for the set.	P I U H	K3	1a		7.0	(C)Cardinality (C)Numeration
The student knows that counting is matching an ordered number name to each member of a set.	P I	K2	1a		7.0	(C)Numeration
The student is able to count members of a set by naming a one-to-one correspondence between number names and members (elements) of a set.	P I	P31 P34 P35	1a		7.0	(C)Numeration
The student is able to count by multiples (e.g., by 2's, 5's, 10's).	P I	P31 P34 P35	1a 1c 1e			(C)Numeration
The student is able to abstract the cardinal number of certain sets without counting.	P I	P33 P37	1a	5a	7.0	(C)Cardinality
The student knows that equivalent sets are those whose members can be placed in a one-to-one correspondence.	P I U H	K2 K5	1a		7.0	(C)Equivalence
The student knows that "greater than" and "less than" are relations defined for the cardinal numbers of sets.	P I	K2 K3	1a 1c		7.0	(C)Ordinality

1. Number Systems
1.1 Whole Numbers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.16 Ordinal Use						
The student knows that an ordinal number refers to the position of one object in an ordered set.	P I U H	K2 K6	1a			(C)Ordinality
The student knows the vocabulary used to describe the position of a number in an ordered set (e.g., first, second, third).	P I U H	K1 K6	1a			(C)Ordinality
The student is able to use ordinal numbers to develop a one-to-one correspondence between ordered sets.	P I	P44 P65	1a	5a		(C)Numeration
The student is able to match the ordinal number name associated with each cardinal number.	P I U	P44 P65	1a			(C)Numeration

1. Number Systems
1.1 Whole Numbers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.17 Ordering						
The student knows the meaning of those symbols associated with ordering (including $<$, $>$, $\neq$, $\leq$, $\geq$).	P I U H	K2	1a			(C)Ordinality
The student knows that for whole numbers a and b, $(a \neq b)$, a is greater than b if b can be subtracted from a (i.e., $6 > 4$ since $6 - 4 = 2$).	P I	K2 K7 K9	1a 1c			(C)Ordinality
The student knows that for whole numbers a and b, $a > b$ if and only if $a - b > 0$.	U H	K2 K9	1a 1c			(C)Ordinality
The student knows that for any two whole numbers a and b, exactly one of the following is true: $a = b$, $a < b$, or $a > b$ (Trichotomy Law).	P I U H	G1 K3	1c			(C)Ordinality
The student knows that the set of whole numbers, when represented on a number line, are in ascending order of magnitude from left to right.	P I U	K2 K4	1a 2a			(C)Ordinality
The student is able to order a set of whole numbers.	P I U H	P34 P65	1c	5a		(C)Ordinality

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.2 Integers						
The student knows that the set of integers is the union of the set of whole numbers and their opposites $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$.	I U H	K2 K5	1a			(C) Structure, math
The student knows that any integer is positive, negative, or zero.	I U H	K3	1a			
The student knows that the set of integers is not dense since between two consecutive integers there does not exist another integer.	I U H	K3 K9	1a			
The student knows that the symbols "+" and "-" may be used to name integers as well as to indicate mathematical operations.	I U H	K2	1a			
The student knows that a raised + or - sign with a numeral indicates a positive or negative number ($^+6$, positive six; $^-6$, negative six).	I U H	K2	1a			
The student knows that those properties, definitions, and computation facts valid for the set of whole numbers are also valid for the set of integers.	I U H	K3	1a 1d			(C) Structure, math
The student knows the identity elements in the set of integers are 0 for addition, 1 for multiplication.	I U H	K2 K3	1a 1d 1e			
The student knows that two integers are additive inverses (opposites) of each other when their sum is zero.	I U H	K2 K3 K9	1a 1d			
The student knows that zero is its own additive inverse.	I U H	K3	1a 1d			

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.2 Integers (Cont.)						
The student knows that on a number line representing the set of integers, the negative integers decrease from zero to the left and the positive integers increase from zero to the right.	I U H	K3 K4 K6	1a 2a			(C)Ordinality
The student is able to locate the set of integers on a number line.	I U H	P31 P44	1a 2a			
The student is able to represent mathematical operations with integers on a number line.	I U	P31 P44 P66	1a 1d			

1. Number Systems
1.2 Integers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.21 Addition						
The student knows that in the set of integers addition is closed, commutative and associative.	I U H	K3	1d			(C)Structure, math
The student knows that for every integer, a, there exists an integer, ^{-}a, called its additive inverse such that $a + {}^{-}a = 0$.	I U H	K2 K9	1a			
The student knows that the sum of two integers, one positive and one negative, is the difference of their absolute values and has the sign of the integer with the greater absolute value.	I U H	K7	1d 1e			
The student knows that the sum of two positive integers is the sum of their absolute values and is positive.	I U H	K7	1d 1e			
The student knows that the sum of two negative integers is the sum of their absolute values and is negative.	I U H	K7	1d 1e			
The student is able to add integers.	I U H	P35	1b 1e	5a		

1. Number Systems
1.2 Integers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.22 Subtraction						
The student knows that subtracting an integer is the same as adding its opposite (additive inverse).	P I	K2 K7	1a 1d 1e			(C) Structure, math
The student knows that the difference, a - b, of two integers a and b, is the sum of a and the opposite (additive inverse) of b.	U H	K7	1a 1d 1e			
The student knows that subtraction is closed in the set of integers.	I U H	K3	1a 1d			
The student knows that subtraction of integers is not commutative or associative.	I U H	K3	1d			
The student is able to subtract integers	I U H	P35	1d 1e	5a		

1. Number Systems
1.2 Integers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words.
1.23 Multiplication						
The student knows that multiplication of integers is closed, commutative, associative, and distributive over addition.	P I U H	K3	1d 1e			(C) Structure, math
The student knows that the product of two integers, both positive or both negative, is a positive integer.	I U H	K2 K7	1b 1e			
The student knows that the product of two integers, one positive and one negative, is a negative integer.	I U H	K2 K7	1b 1e			
The student knows that for a and b integers, the product a · b is zero if and only if a or b is zero.	P I U H	K2 K7 K9	1d 1e			
The student is able to multiply integers.	I U H	P35	1e	5a		

1. Number Systems
1.2 Integers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.24 Division						
The student knows that in the set of integers division is not closed, associative, or commutative.	I U H	K3	1d			(C)Structure, math
The student knows that in the set of integers, division by zero is undefined.	I U H	K3	1b 1d			
The student knows that the quotient of two integers, both positive or both negative, is positive.	I U H	K7	1b 1e			
The student knows that the quotient of two integers, one positive and one negative, is negative.	I U H	K7	1b 1e			
The student is able to divide integers.	I U H	P35	1b 1e	5a		

1. Number Systems
1.2 Integers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Headings	Taxonomy (C) Concept/ (V1, V2) Value Words
1.25 Absolute Value						
The student knows that $\|x\|$ is the symbol for the absolute value of x.	I U H	K2	1a			
The student knows that every integer has an absolute value and it is non-negative.	I U H	K2	1a			
The student knows that the absolute value of an integer if non-negative is that integer, and if negative, the opposite of that integer.	I U	K2 K9	1a 1e			
The student knows that $\|x\|$, the absolute value of an integer x, is x if $x \geq 0$ or -x if $x < 0$.	H	K2 K9	1a 1e			
The student knows that the absolute value of an integer may be represented as the distance from zero on a number line.	I U H	K2	1a 2a			
The student is able to perform operations or simplifications involving absolute value of integers.	I U H	P35 P65	1b 1e	5a		

1. Number Systems

1.2 Integers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.26 Order						
The student knows that the integers on a number line are ordered according to the direction and magnitude of their distances from zero.	I U H	K2 K6	1a 2a			(C)Ordinality
The student knows that of two integers represented on a number line, the one on the right is larger.	P I U H	K2 K6 K9	1a 2a			(C)Ordinality
The student knows that for integers a and b, $a > b$ if and only if $a - b = c$ and $c > 0$. ($^-9$ is greater than $^-14$ because $^-9 - {}^-14 = 5$, 5 being positive)	I U H	K2 K9	1c			(C)Ordinality
The student knows that the Trichotomy Law is valid in the set of integers (i.e., for integers a, b: $a < b$, $a > b$, or $a = b$).	U H	K3	1c			
The student is able to order integers.	I U H	P34	1c	5a		(C)Ordinality

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Headings	Taxonomy (C) Concept/ (V1, V2) Value Words
1.3 Rational Numbers, Positive, Zero and Negative						
The student knows that a rational number is any number that can be named in the form $\frac{a}{b}$, $b \neq 0$, when a and b are integers.	I U H	K2	1a		2.5	(C) Number relationships
The student knows that the set of whole numbers and the set of integers are subsets of the set of rational numbers.	I U H	K2 K5	1a			(C) Structure, math
The student knows that any integer can be expressed in rational form $\frac{a}{b}$ with $b = 1$.	I U H	K2	1a 1b		2.5	(C) Number relationships
The student knows the three common forms of rational numbers are fraction, decimal fraction (decimal) and percent.	I U H	K2	1a 1b		2.5	
The student knows the definition of key terms associated with rational numbers, including rational number, fraction, numerator, denominator, proper, improper, mixed numerals, repeating decimals, and decimal fractions.	I U H	K1 K2	1a		2.5	
The student knows that the properties, definitions, and computation facts valid for the set of integers are valid for the set of rational numbers.	I U H	G2 K2 K3 K5	1a 1d 1e			(C) Structure, math
The student knows that the set of rational numbers is dense since between any two rational numbers there is another rational number.	I U H	K3 K9	1a 5c			
The student knows the multiplicative inverse of a nonzero rational number b is $\frac{1}{b}$ and that $b \cdot \frac{1}{b} = 1$.	I U H	K2	1a 1d			

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.3 Rational Numbers, Positive, Zero and Negative (Cont.)						
The student knows that the multiplicative inverse (reciprocal) is defined for every rational number except zero.	I U H	K3	1a 1d			
The student knows the identity elements for addition (zero, 0) and multiplication (one, 1) in the set of rational numbers.	I U H	K2	1a 1d			
The student knows that in the set of rational numbers the product of two numbers is zero if and only if one or both is zero.	I U H	K2 K7 K9	1d 1e			
The student knows that the set of rational numbers is an ordered field.	H	K2 K3	1a 2a			(C) Structure, math
The student knows that the set of rational numbers correspond to points on a number line.	I U H	K3 K5	1a 2a			
The student is able to locate the set of rational numbers on a number line.	I U H	P31 P44	1a 2a			
The student is able to represent mathematical operations with rational numbers on a number line.	I U H	P31 P44 P66	1a 1d 2a			
The student knows that a number written in the form $\frac{a}{b}$ with $b \neq 0$ is in fractional form.	I U H	K2 K9	1a		2.5	
The student knows that a numeral in the form $\frac{a}{b}$ represents the quotient of a divided by b. (e.g., $\frac{3}{4} = 3 \div 4$)	I U H	K2	1a 1d		2.5	

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.3 Rational Numbers, Positive, Zero and Negative (Cont.)						
The student knows that a fractional number has an infinite set of equivalent fractional names. ($\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} \dots$)	P I U H	K3	1b 1c		2.5	(C) Equivalence
The student knows equivalent fractions may be produced by multiplying by an appropriate form of one.	I U H	K7	1b 1c		2.5	
The student is able to express a rational number greater than one as an improper fraction or a mixed numeral.	I U H	P35 P66	1b	5a	2.5	
The student knows that in a decimal fraction the decimal point separates the fractional number from the whole number.	I U	K2 K7	1a		2.2	
The student knows the place value represented by each digit in a decimal fraction.	I U H	K3 K6	1a		2.2	
The student is able to express a decimal numeral in expanded notation. (e.g., $7.125 = \frac{7}{1} + \frac{1}{10} + \frac{2}{100} + \frac{5}{1000}$)	I U H	P35 P66	1a 1b		2.2	
The student knows the forms of a decimal fraction (terminating and nonterminating, repeating and nonrepeating).	I U H	K3	1a		2.2	
The student knows that any repeating or terminating decimal represents a rational number.	I U H	K2 K5	1a 1b		2.2	
The student knows a method to write any rational number in either fractional or decimal form.	I U H	K2 K7	1a 1c			

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.3 Rational Numbers, Positive, Zero, and Negative (Cont.)						
The student knows the equivalent forms of commonly used rational numbers ($\frac{1}{4} = .25 = 25\%$, $\frac{1}{10} = .1 = 10\%$).	I U H	K3 K5	1a 1b		2.5	
The student is able to perform the fundamental operations on rational numbers, including fractions and decimal numerals.	I U H	P35	1e	5a		

1. Number Systems
1.3 Rational Numbers, Positive, Zero and Negative

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.31 Addition						
The student knows that the sum of two rational numbers, $\frac{a}{b}$ and $\frac{c}{d}$, is $\frac{ad}{bd} + \frac{cb}{db} = \frac{ad + bc}{bd}$. (e.g., $\frac{3}{4} + \frac{5}{7} = \frac{3 \cdot 7}{4 \cdot 7} + \frac{5 \cdot 4}{7 \cdot 4} = \frac{3 \cdot 7 + 4 \cdot 5}{4 \cdot 7} = \frac{21 + 20}{28} = \frac{41}{28}$)	U H	K2	1e			(C) Number operation
The student knows that in the set of rational numbers addition is closed, commutative and associative.	I U H	K3	1d			(C) Structure, math
The student is able to find a common denominator (multiple) for two or more rational numbers in fractional form.	I U H	P35	1b	5a		
The student is able to rename fractions with different denominators as fractions with the same denominator.	I U H	P35 P66	1b			
The student knows that the sum of two fractions with the same denominator is the sum of the numerators divided by that denominator (i.e., $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$).	I U H	K2	1e			
The student is able to add rational numbers in fractional form (including fractions and mixed numbers with the same or different denominators).	I U H	P35	1b 1e	5a		
The student knows the rules for addition of rational decimal fractions (including addition of equal place values and placement of the decimal point).	I U H	K2	1e			
The student is able to add rational decimal fractions.	I U H	P35	1e	5a		

1. Number Systems
1.3 Rational Numbers, Positive, Zero, and Negative

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.32 Subtraction						
The student knows that the set of rational numbers is closed for subtraction.	I U H	K3	1d			(C) Structure, math
The student knows that subtraction in the set of rational numbers is not commutative or associative.	I U	K3	1a 1d			(C) Structure, math
The student knows that every rational number has an additive inverse (opposite).	U H	K2	1b			
The student knows the rules for addition of rational numbers that apply to subtraction in the set of rational numbers (including the inverse nature of the operations, place value rules, regrouping or borrowing, placement of the decimal point, and the common denominator properties).	I U H	K2	1e			
The student is able to subtract rational numbers (including fractions and decimals).	I U H	P35	1e	5a		

1. Number Systems
1.3 Rational Numbers, Positive, Zero, and Negative

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.33 Multiplication						
The student knows that the product of two rational numbers is the product of the numerators divided by the product of the denominators (i.e., for rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$).	I U H	K2	1b 1e			
The student knows that in the set of rational numbers multiplication is closed, commutative, associative, and distributes over addition.	I U H	K3	1a 1d			(C) Structure, math
The student knows that factors in mixed numeral form can be renamed as improper fractions.	I U H	K2	1b 1e			
The student knows that the product of two fractions is a fraction $\frac{a}{b}$, where a is the product of the numerators and b the product of the denominators.	I U H	K7	1a 1d			
The student is able to multiply rational numbers in fractional form (including fractions and mixed numerals).	I U H	P35	1b 1e	5a		
The student knows the rules for multiplication of rational decimal fractions (including placement of the decimal point, regrouping or carrying and place value rules).	I U H	K2	1d 1e			
The student is able to multiply rational decimal fractions.	I U H	P35	1b 1e	5a		

1. Number Systems
1.3 Rational Numbers, Positive, Zero, and Negative

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.34 Division						
The student knows that in the set of rational numbers division may be defined as multiplication by the multiplicative inverse (reciprocal) (i.e., $\frac{a}{b} = a \cdot \frac{1}{b}$).	I U H	K2	1a 1d			
The student knows that every rational number except zero has a multiplicative inverse.	I U H	K3	1a 1d			
The student knows that division in the set of rational numbers is closed except for division by zero which is undefined.	I U H	K3	1a 1d			(C) Structure, math
The student knows that in the set of rational numbers division is not commutative or associative.	I U H	K3	1a 1d			(C) Structure, math
The student knows the properties of multiplication of fractions that apply to division of fractions (including the inverse nature of the operations and the use of improper fractions to replace mixed numerals).	I U H	K3	1b 1e			
The student knows the rules for division of rational decimal fractions (including placement of the decimal point and place value rules).	I U H	K2	1b 1e			
The student is able to divide rational numbers (including fractions and decimals).	I U H	P35	1b 1e	5a		

1. Number Systems

1.3 Rational Numbers, Positive, Zero, and Negative

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.35 Absolute Value						
The student knows that $\lvert x \rvert$ is the symbol for the absolute value of x.	U H	K2	1a			
The student knows that every rational number has an absolute value and is non-negative.	I U	K2	1a			
The student knows that the absolute value of a rational number, if non-negative, is that number, and if negative, the opposite of that number.	I U	K2 K9	1a 1e			
The student knows that $\lvert x \rvert$, the absolute value of a rational number x, is x if $x \geq 0$ or $-x$ if $x < 0$.	H	K2 K9	1a 1e			
The student knows that the absolute value of a rational number may be represented as a distance from zero on a number line.	I U H	K2	1a 2a			
The student is able to perform operations and simplifications involving absolute value of rational numbers.	I U H	P35	1b 1e	5a		

1. Number Systems
1.3 Rational Numbers, Positive, Zero, and Negative

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.36 Order						
The student knows that for two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ $(b, d \neq 0)$ that $\frac{a}{b} > \frac{c}{d}$ if and only if $ad > bc$.	U H	K2 K9	1c			(C) Ordinality
The student knows that for two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ $(b, d \neq 0)$ that $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.	U H	K2 K9	1c			(C) Ordinality
The student knows a method to compare decimal fractions.	I U H	K5 K7	1c			(C) Ordinality
The student knows that equivalent forms of rational numbers may be used as an aid to order a set of rational numbers in different forms.	U H	K2 K7	1c			(C) Equivalence (C) Ordinality
The student is able to order rational numbers including fractions, decimals, and mixed forms.	I U	P34	1c	5a		(C) Ordinality

I. Number Systems

1.3 Rational Numbers, Positive, Zero, and Nagative

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.37 Ratio, Proportion, and Percent						
The student knows other symbols for a ratio $\frac{a}{b}$ (including a : b, a to b).	I U H	K2	1b		2.5	
The student knows a ratio is a comparison of two numbers by division (i.e., the ratio of a compared to b is $\frac{a}{b}$).	I U H	K2 K5	1a 1b 1d		2.5	
The student knows that a percent is defined as a ratio with denominator of one-hundred and is denoted by the symbol % (e.g., 50% is $\frac{50}{100}$).	I U H	K1 K2	1a			
The student is able to rename any rational number as a percent.	I U H	P35	1b 1e	5a	2.5	
The student is able to rename a number in percent form as either a fraction or a decimal.	I U H	P35	1b 1e	5a	2.5	
The student knows the rules for computing with percent (e.g., converting the percent to a fraction or a decimal).	I U H	K2 K3	1b 1e		2.5	
The student knows the meaning of the key terms associated with percent (including base, rate, and percentage).	I U H	K1 K2	1a		3.4	
The student knows that base x rate = percentage.	I U H	K2	1d	5a	3.4	
The student is able to compute percentage.	I U H	P35	1d	5a		
The student knows areas of application for percent (e.g., banking, commerce, statistics, and communications).	I U H	K6	4b		3.4	
The student knows that a proportion is a statement of equality between two ratios.	I U	K2	1a 1c			

1. Number Systems
1.3 Rational Numbers, Positive, Zero, and Negative

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.37 Ratio and Proportion (Cont.)						
The student knows the key terms associated with proportion (including means, extremes, mean proportional, terms of a proportion, and constant of proportionality).	I U H	K1 K2	1a			
The student knows the equivalent forms of a proportion including $\frac{a}{b} = \frac{c}{d}$, $\frac{a}{c} = \frac{b}{d}$, $\frac{c}{a} = \frac{d}{b}$, and $\frac{b}{a} = \frac{d}{c}$.	I U H	K2 K3	1c 1d			
The student knows that in a proportion the product of the means is equal to the product of the extremes (i.e., for b and $d \neq 0$, $\frac{a}{b} = \frac{c}{d}$ implies $ad = bc$).	U H	K2 K7	1a 1e			
The student is able to express equal products as a proportion (i.e., $ab = cd$ can be written $\frac{a}{c} = \frac{d}{b}$).	U H	P35 P44	1c	5a		
The student is able to find the solution set for a variable term in a proportion.	I U H	P35 P63	1e	5a		
The student is able to express any ratio as a ratio with a denominator of one.	U H	P35 P44	1a		2.5	
The student knows the definitions of the major kinds of mathematical variations (including direct, indirect or inverse, joint, and combined).	U H	K2	1a 1d			
The student knows the form of proportion that indicates the types of variations (e.g., x varies directly as y is written $x = ky$ or $\frac{x_1}{x_2} = \frac{y_1}{y_2}$).	U H	K3	1c 1d			
The student knows types of problems that can be translated to proportion (including scale drawings, unit cost, motion, and percent).	I U H	K6 K8	2a 4b	5a		

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.4 Real Numbers						
The student knows that the set of real numbers consists of the union of the set of rational numbers and the set of irrational numbers.	U H	K3 K5	1a			(C) Structure, math
The student knows that the field and order properties apply to the real numbers.	U H	K3	1a			(C) Structure, math
The student is able to use the field and order properties in solving problems involving real numbers.	U H	P35 P44 P63	1e	5a		
The student knows that there is a one-to-one correspondence between the real numbers and the points on the number line (i.e., for every point on the line there is a real number).	U H	K3 K5	2a			
The student is able to represent the real numbers on a number line.	U H	P31 P44 P66	1a 2a			(C) Number relationship
The student knows that an irrational number is one that cannot be written as the quotient of an integer and a counting number.	U H	K2	1a			
The student knows that numbers named by nonrepeating, nonterminating decimal numerals are called irrational numbers (e.g., $\log 2 = .30103\ldots$, $\pi = 3.14159\ldots$, $\sqrt{3} = 1.732\ldots$, $\sin 37^{\circ} = .6018\ldots$, $e = 2.71828\ldots$)	U H	K1 K2	1a			
The student knows that if the square root of a whole number is not a whole number then it is an irrational number.	U H	K2 K9	1a			

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.4 Real Numbers (Cont.)						
The student is able to determine if his estimates of roots of whole numbers are reasonable.	U-H	P24 P36 P47	1d 1e	5a		
The student is able to follow an argument that proves that a number is irrational.	H	P47 P53 P62	6a 6d			

1. Number Systems
1.4 Real Numbers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.41 Operations						
The student is able to perform computations with real numbers.	U H	P35	1e 4a	5a		
The student knows which operations of irrational numbers, or an irrational number and a real number, result in irrational answers.	H	K5 K7	1a			
The student knows that "rationalizing the denominator" means transforming a fraction with a denominator involving radicals into an equivalent fraction which has a denominator free of radicals.	U H	K2	1a 1b			
The student is able to simplify expressions containing irrational numbers.	U H	P35 P37	1a 1b	5a		
The student is able to name the approximate square root of a number (including using an algorithm, estimating, tables, linear interpolation, logarithms).	U H	P35	1e 4a	5a		
The student is able to solve problems involving square root (including the use of the Pythagorean Property, solving quadratic equations, using the law of cosines).	U H	P35 P63	1e 1b	5a		

1. Number Systems
1.4 Real Numbers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.42 Absolute Value						
The student knows that the absolute value for all real numbers is defined as: $\|x\| = x$ if $x \geq 0$ and $\|x\| = -x$ if $x < 0$.	U H	K2	1a			
The student is able to perform computation with absolute values of real numbers.	U H	P35	1c 1e	5a		
The student knows that the absolute value of a real number may be represented as a distance from zero on the number line.	U H	K2	1a			
The student is able to use the properties of absolute value from the set of rational numbers in the set of real numbers.	U H	P35	1a 1e			

1. Number Systems
1.4 Real Numbers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.43 Order						
The student knows that between any two real numbers there exists at least one real number (i.e., the set of real numbers is dense).	U H	K3 K6	1a			
The student is able to order the set of real numbers using the comparison axiom (Law of Trichotomy).	U H	P34	1a 1c			
The student knows that when comparing two real numbers on a number line the number on the right is always the larger of the two.	U H	K2 K6	1a 1c			(C) Ordinality
The student knows that the transitive property of order is valid in the set of real numbers (i.e., if a $<$ b and b $<$ c then a $<$ c).	U H	K3	1a 1c			(C) Ordinality

1. Number Systems
1.4 Real Numbers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.44 Limits and Sequences						
The student knows a definition of and the notation used to represent a sequence.	U H	K2	1a			
The student knows that a finite sequence is a set of elements that can be placed in one-to-one correspondence with a proper subset of the positive integers.	U H	K2	1a			
The student knows that an infinite sequence is a set of elements that can be placed in one-to-one correspondence with the set of positive integers.	U H	K2	1a			(C) Infinity, math
The student knows that an arithmetic sequence is any sequence in which each term after the first is obtained by adding a fixed number, called the common difference, to the preceding term.	U H	K2	1a			
The student knows that a geometric sequence is any sequence in which each term after the first is the product of the preceding term and a fixed number, called the common ratio.	U H	K2	1a			
The student is able to recognize geometric or arithmetic sequences by finding their common ratio or common difference.	U H	P33 P47	1a	5a		
The student is able to find missing terms in arithmetic or geometric sequences.	U H	P34 P35 P63	1b	5a		
The student is able to find the n^{th} term of an arithmetic or geometric sequence.	U H	P34 P35 P63	1a	5a		

1. Number Systems
1.4 Real Numbers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.44 Limits and Sequences (Cont.)						
The student knows that a series is the indicated sum of the terms of a sequence (the sequence $a_1, a_2, \ldots a_n$ yields the series $a_1 + a_2 + \ldots + a_n$).	U H	K2	1a			
The student knows the rules for finding the sum of n terms of arithmetic and geometric series.	U H	K2 K7	1a			
The student is able to name the sum indicated by n terms of an arithmetic or geometric series.	U H	P35	1a			
The student is able to express a series in either summation notation or expanded form (i.e., $\sum_{i=1}^{5} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$)	U H	P35 P37	1a	5a		
The student knows the summation definition for the binomial theorem (i.e., n: positive whole number, a: real, $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$)	H	G1 K2 K3	1a 4b			
The student knows the meaning of the terms associated with infinite sequences including convergence, divergence, bounds, increasing, decreasing, partial sums.	U H	K1 K2	1a			
The student knows definition of limit (including limit of a sequence and limit of a function).	H	K2	1a			
The student is able to compute the limit of convergent sequences.	H	P35	1a	5a		

1. Number Systems
1.4 Real Numbers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.44 Limits and Sequences (Cont.)						
The student knows the theorems on limits for sequences and functions including multiplication by a constant and the sums and products.	H	K7	1a			(C) Limits
The student knows a definition for a sequence of partial sums (i.e., $s_1 = a_1, s_2 = a_1 + a_2, s_3 = a_1 + a_2 + a_3$ for the sequence $s_1, s_2, \ldots, s_n$).	H	K2	1a			
The student is able to find a rational number equivalent to a repeating decimal by computing the limit of a sequence of partial sums.	H	P35 P63	1a	5a		
The student knows the Δx (delta-x) method for finding average and instantaneous rates of change.	H	K7	1a			
The student knows the notation used to indicate differentiation and integration (f^1 the first derivative of f, $\int_1^3 x^2dx$ the integral of x^2 in the interval 1 to 3).	H	K2	1a			
The student is able to use the Δx (delta-x) method to find the instantaneous rate of change (derivative).	H	P35 P63	1a	5a		
The student knows formulas for taking derivatives of various algebraic, exponential, and trigonometric expressions.	H	K2 K7	1a			
The student is able to use derivatives to solve problems involving: maxima, minima, inflection points, and the finding of velocity or acceleration.	H	P35 P63	1a 4b	5a		

1. Number Systems
1.4 Real Numbers

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.44 Limits and Sequences (Cont.)						
The student knows definitions for the mathematical integral.	H	K1 K2	1a			
The student knows that taking derivatives and finding integrals are inverse operations.	H	K3 K5	1a 1d			
The student knows formulas for finding integrals of various algebraic, exponential, logarithmic, and trigonometric expressions.	H	K2 K7	1a			
The student is able to use integrals to solve problems such as: areas or volumes under curves and those proceeding from acceleration to velocity or distance.	H	P35 P63	1a 4a 4b 5a	5a		

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.5 Complex Numbers						
The student knows that the set of real numbers is a subset of the set of complex numbers.	H	K3 K5	1a			(C) Structure, math
The student knows a definition of a complex number.	H	K2	1a			
The student knows the definition of the symbol "i" for imaginary numbers.	U H	K1 K2	1a			
The student knows the forms in which a complex number may be expressed including the standard form a + bi.	H	K2	1b 1e			
The student is able to represent a complex number in several forms (e.g., standard form, polar form and vector form (ordered pairs), exponential form).	H	P35 P66	1b 1e	5a	1.7	
The student knows that a rectangular coordinate system may be used as a geometric model for the set of complex numbers by assigning the pure imaginary numbers to the y axis and the real numbers to the x axis.	H	K7	1a 5c			(C) Dimensionali
The student is able to show the graphic representation of a complex number in a rectangular coordinate system	H	P35 P44 P66	1a 5c	5a	5.3	(C) Dimensionali
The student knows a definition for equality of complex numbers.	H	K2	1a			
The student knows the definitions for the basic operations (addition, subtraction, multiplication, and division) for complex numbers.	H	K2 K7	1b 1d			(C) Operations
The student is able to find the absolute value of a complex number (e.g., $\lvert a + bi \rvert = \sqrt{a^2 + b^2}$).	H	P35 P63	1a	5a		

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.5 Complex Numbers (Cont.)						
The student knows that the field properties are valid for the set of complex numbers.	H	K3 K7	1a 1d		1.0 1.7	(C) Structure, math
The student is able to verify the field properties of the complex number system.	H	P47	1a 1d			(C) Structure, math
The student knows the definitions of the conjugate of a complex number.	H	K2	1a			
The student is able to perform operations with complex numbers and their conjugates.	H	P35	1b 1d	5a		
The student is able to find the n^{th} power of a complex number using DeMoivre's Theorem.	H	P35	1b 1d 1e			

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.6 Matrices and Determinants						
The student knows that a matrix is a rectangular array of numbers.	H	K2	1a			
The student knows that a determinant is a number associated with each square matrix.	H	K2	1b			
The student knows the rule for finding the determinant of a two by two matrix.	H	K2 K7	1a 1d			
The student knows the meaning of the basic terms associated with determinants (including order, element, minor and cofactor).	H	K1 K2	1a			
The student knows that the value of a determinant can be found by choosing a row or column and finding the sum of the products of each element and its cofactor.	H	K2 K7	1a 1e			
The student is able to compute the determinant of each square matrix.	H	P35	1e	5a		
The student knows the rules for operations on rows and columns of a square matrix used in evaluating determinants.	H	K2 K7	1d			
The student knows that when two rows or columns of a square matrix are the same, then the determinant is zero.	H	K3	1a 1b			
The student is able to use Cramer's Rule to solve systems of equations.	H	P35 P63	1c 1e	5a		
The student knows that the determinant of the product of matrices is equal to the product of the determinants of the matrices.	H	K3 K5	1d			

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.6 Matrices and Determinants (Cont.)						
The student knows that a matrix of m rows and n columns has order (dimension) m by n.	H	K2	1a			
The student knows that a square matrix is a matrix with n rows and n columns and has order n.	H	K2	1a			
The student knows that two matrices are equal if and only if they have the same order (dimensions) and all corresponding elements are equal.	H	K2	1a 1c			
The student knows the symbolism of matrices (e.g., $A_{(i,\ j)}$ may indicate the element in row i and column j of matrix A).	H	K2				(C) Symbols
The student knows that a zero (or null) matrix is one with all elements zero.	H	K2	1a 1d			
The student knows the definition for addition of matrices.	H	K2 K7	1d			
The student knows that the identity element for addition of matrices is the zero matrix.	H	K2	1b 1d			
The student knows the definition for the product of a scalar and a matrix.	H	K2 K7	1d			
The student knows the definition for multiplication of matrices.	H	K2 K7	1b 1d			
The student knows that the product of two nonzero matrices can be the null matrix.	H	K3 K8	1b 1d			
The student is able to multiply and add matrices.	H	P35	1b 1d	5a		

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.6 Matrices and Determinants (Cont.)						
The student knows the properties of the identity matrix for multiplication (including diagonal pattern, commutivity and dimension).	H	K3	1b 1d			
The student knows that a square matrix is singular if it has a zero determinant and nonsingular if it has a nonzero determinant.	H	K2 K3 K9	1a 1b			(C) Singularit
The student knows that a square matrix has a multiplicative inverse if it is nonsingular.	H	K3 K9	1b 1d			
The student is able to find the multiplicative inverse of a square matrix.	H	P33 P35 P44	1b 1d			
The student is able to represent a system of equations with a matrix.	H	P44 P66	1c 4b	5a		
The student knows the row, column transformation rules for a matrix that represents a system of equations.	H	K2 K7	1b 4b		3.3	(C) Structure, math
The student is able to use matrices to solve systems of equations.	H	P35 P63	1b 1c 1d 4b	5a	3.3	

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.7 Vectors						
The student knows that vectors are directed line segments used to represent quantities having magnitude and direction (vector quantities).	U H	K2	1a 9b			
The student knows the terminology used with vectors (including origin, initial point, terminal point, unit vector, zero or null vector, vector space).	H	K1 K2	1a 9b			
The student knows the symbols common to vectors (including $\overline{a}$, the vector a, x for cross product, · for dot product)	H	K2	1a 9b			(C) Symbol
The student knows the standard representation of a vector.	H	K2	1a 9b			
The student knows that equivalent vectors have the same magnitude and direction.	H	K2	1a 9b			(C) Equivalence
The student knows that scalar quantities are real numbers having only magnitude.	H	K2	1a 9b			
The student knows the resultant of a vector multiplied by a scalar.	H	K2	1d			
The student knows the properties of a vector multiplied by a scalar.	H	K2 K3				
The student knows that nonzero vectors in the same direction or in opposite directions are parallel.	H	K2 K3	1a 5c		5.3	
The student knows that a nonzero vector a is parallel to a nonzero vector b if and only if vector a can be expressed as a scalar multiple of vector b.	H	K2 K3	1a 9b		5.3	

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.7 Vectors (Cont.)						
The student knows that the sum (resultant) of two vectors is a uniquely determined vector.	H	K2 K3	1a 9b		5.3	
The student is able to add and subtract two vector quantities.	H	P35	1d 5c	5a	5.3	
The student knows that vectors can be represented as the resultant of an x-component and a y-component which are parallel to x and y axis respectively.	H	K2 K3	1a 5c 9b		5.3	
The student is able to represent addition and subtraction of vectors graphically.	H	P31 P35 P44 P66 P76	1d 5c	5a	5.3	
The student knows that vectors form a group with respect to addition.	H	K3	1a			(C) Structure, math
The student knows a definition for the inner product (dot product) of two vectors.	H	K2 K7	1d 5c			
The student knows that the inner product (dot product) of vectors has commutative, associative, distributive, and substitution properties.	H	K3	1d			
The student knows that the inner product of a vector with itself is equal to the square of its magnitude.	H	K2 K3	1a 9b			
The student is able to determine the magnitude (norm) of a vector.	H	P35 P63	1a 5c	5a	5.3	

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.7 Vectors (Cont.)						
The student knows that the dot product (inner product) may be used to study work and force.	H	K6 K8	1d 4b			
The student knows that two nonzero vectors are said to be perpendicular to each other if and only if their inner product (dot product) is zero.	H	K2 K3 K9	1a 1c		5.3	
The student knows a definition of vector product (cross product).	H	K2	1a			
The student knows that the vector product (cross product) of vectors has distributive, associative, and zero product properties (a vector in cross product with itself gives a zero vector).	H	K3	1d			
The student knows that the vector product (cross product) of vectors has an anticommutative property (i.e., one product is the additive inverse of the other).	H	K3	1a 1d			
The student knows that the vector product (cross product) can be calculated using determinants.	H	K7	1a			
The student is able to rename a vector with either polar or rectangular coordinates.	H	P31 P35 P66	1a 1b	5a		
The student knows that any ordered pair or triple of real numbers, (a, b) or (a, b, c), can represent a vector.	H	K2	1a 5c		5.3	
The student knows the meaning of "right-handed system" as it refers to coordinate systems in three dimensional space.	H	K2			5.3	

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.7 Vectors (Cont.)						
The student knows that any vector A may be expressed as a combination of the nonzero, nonparallel, coplanar vectors B and C, i.e., vector A is equal to a scalar times B plus a scalar times C.	H	K2	1c		5.3	
The student knows that directions in space can be stated with direction angles, direction cosines, and direction numbers.	H	K7	5c		5.3	(C) Space
The student knows areas of study that use vectors, including applications of plane, projective, and space geometry.	H	K6 K7	4b		5.3	

1. Number Systems

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.8 Algebraic Expressions						
The student knows definitions for the terms associated with algebraic expressions (including coeffecient, constant, degree, term, polynomial, quadratic, complex fractions, leading term, linear polynomial).	U H	K1 K2	1a			
The student knows that an expression is a combination of numerals and/or variables connected by operational symbols.	U H	K2	1a			
The student knows that an expression names a specific number or a set of numbers.	U H	K3	1a			
The student is able to evaluate an algebraic expression by numerical substitution.	U H	P24 P35	1a 1b			

1. Number Systems
1.8 Algebraic Expressions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.81 Polynomials						
The student knows the meaning of the terms associated with polynomials (including monomial, binomial, trinomial, polynomial degree, term, coefficient).	U H	K1 K2	1a			
The student knows the ways in which polynomials are classified (including number of terms or variables, by degree and by the nature of the coefficients).	U H	K3 K5	1a			
The student knows a standard form for expressing a polynomial (e.g., $a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$ ascending order of terms).	U H	K2	1a 1b			
The student knows that the set of polynomials over the rational numbers forms a field for the operations of addition and multiplication.	H	K3 K5	1a 1d			(C) Structure, math
The student is able to do the basic operations (add, subtract, multiply and divide) with polynomials.	U H	P35	1b 1d 1e	5a		
The student knows that a prime polynomial is any nonfactorable polynomial other than 1, 0, and -1.	U H	K2	1a 1b			
The student knows that "factoring a polynomial over a set" is defined to be expressing the polynomial as an indicated product of polynomials over the set.	U H	K2	1b 1e			
The student knows the applications of the distributive law to factoring and multiplying polynomials.	U H	K6	1b 1d 1e			

I. Number Systems
1.8 Algebraic Expressions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.81 Polynomials (Cont.)						
The student is able to factor some polynomials by inspection (including perfect square trinomials, difference of two squares, and the sum or difference of two cubes).	U H	P35	1b 1d 1e	5a		
The student is able to factor nonprime, second degree polynomials of the form $ax^2 + bx + c$ and $ax^2 + bxy + cy^2$ over a set.	U H	P35	1b 1d 1e	5a		
The student is able to determine a prime factorization of a polynomial over the set of integers or the set of rationals.	U H	P35 P44	1b 1d 1e	5a		
The student is able to verify factors of a polynomial using synthetic division or multiplication.	U H	P35 P43 P47	1b 1e	5a		
The student knows the Remainder Theorem.	H	G1 K7	1b 1e			
The student is able to apply the Remainder Theorem to evaluate a polynomial for any real number.	H	P35 P63	1b 1e	5a		
The student knows the Factor Theorem.	H	G1 K7	1b 1e			
The student is able to apply the Factor Theorem to find and verify factors of a polynomial.	H	P35 P53	1b 1e	5a		
The student is able to recognize the pattern exhibited by coefficients and exponents when binomials are expanded.	U H	P43 P46	1b 1e			
The student knows the Binomial Theorem.	H	K2 K7	1b 1e			

1. Number Systems
1.8 Algebraic Expressions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.81 Polynomials (Cont.)						
The student is able to expand binomials using the Binomial Theorem.	H	P35	1b 1d 1e	5a		
The student knows a relationship between the coefficients in the binomial expansion and Pascal's triangle.	H	K5 K8	1a 1b			
The student knows applications of the Binomial Theorem to problems in genetics and probability.	H	K6 K7	4b			
The student knows the definition of a polynomial function.	H	K2	1a			

1. Number Systems
1.8 Algebraic Expressions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
1.82 Rational Expressions						
The student knows that a rational expression is the quotient of two polynomials $P_1 \div P_2$ where P_2 is not the zero polynomial.	U H	K2	1a			
The student knows a rational expression is not defined when the denominator has value 0.	U H	K2 K9	1a			
The student knows algorithms for the operations of addition, subtraction, multiplication, and division of algebraic rational expressions.	U H	K7	1b 1d 1e			
The student knows that a rational expression is in simplest form if the numerator and denominator do not have a common factor other than 1 or -1.	U H	K2	1b 1e			
The student is able to simplify rational expression by dividing numerator and denominator by the greatest common factor.	U H	P35 P37	1b 1e	5a		

2. Numeration

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
2. Numeration						
The student knows the location and use of print and nonprint materials related to numeration in mathematics (e.g., card catalog: "Classification, Decimal," "Rational Number," "numbers, Theory of"; Reader's Guide: "Mathematical Recreations," "Numbers, Theory of"; Area and Building Audio-visual Catalogs: "Mathematics, Base Concept," "Mathematics, Decimal," "Mathematics, Rational Number."	I U H	K6	1a 1b 1c 1d 1e 1f 1g 1h			(C) Resources, mathematica (V1) Inquiry
The student knows that a numeral is a symbol used to name a number.	P I U H	K1 K2	1a 3		1.15	(C) Symbols
The student knows the idea of number is independent of symbol.	P I U	K2 K3 K5	1a 3		1.15	(C) Numeration (C) Symbol
The student knows there have been many sets of numeration symbols (including Hindu-Arabic, Roman, Egyptian).	P I U	K2 K3 K4	3		10.	(C) Numeration
The student knows the definition of cardinal and ordinal numbers.	P I U	K1 K2	1a		1.15 1.16	(C) Numeration
The student knows there are numeration systems that are nonpositional (e.g., Egyptian, hieroglyphics, tallying).	P I U	K1 K2	1b 3		10.	
The student knows that the value of a numeral in a nonpositional system is the sum of the values of the individual symbols.	P I U	K2 K7	1b 3			
The student knows that in a positional numeration system location of a digit determines the value of a numeral.	P I U H	K3 K6	1a 3			
The student is able to determine the advantages and disadvantages of different numeration systems.	P I U	P45	1b 3	5a		(C) Numeration

2. Numeration

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
2.1 Base Systems						
The student knows that a base for a number system may be any whole number greater than one.	I U H	K2 K3	1a 3			(C) Numeration (C) Structure, math
The student knows that in any base the number of counting symbols must equal the number of the base used (e.g., a base ten system requires ten symbols 0, 1, 2, ..., 9).	P I U H	K2 K3	1a			(C) Numeration
The student knows that bases greater than ten require symbols in addition to the Hindu-Arabic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.	U H	K3 K8	1a 3			(C) Numeration
The student knows that the symbol "." (point) used in a numeral separates the whole number part from the fractional part.	P I U H	K2 K6 K7	1a		1.30	
The student knows the ways to signify the base in which a numeral is written (e.g., $64_{(7)}$ or 64_{seven} denotes base seven).	I U H	K2	1a			
The student knows the terms used to name various base systems (e.g., binary, octal, decimal, duodecimal, hexadecimal).	I U H	K1 K3	1a			
The student knows that the value of any position in a numeral is a power of the base (b) and the first position to the left of the "point" has the value b^0 or 1.	I U H	K2 K6 K7	1a 1b			
The student knows that the place values of a numeral are consecutive powers of the base and these powers decrease from left to right.	I U H	K2 K6 K7	1a 1b			(C) Numeration

2. Numeration

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
2.1 Base Systems (Cont.)						
The student knows that the number denoted by any digit in a numeral is the product of the digit and the place value of the position it occupies (e.g., In 235 the 2 means 2×10^2).	I U H	K2 K6 K7	1a			
The student knows cases in which zero is significant in a numeral (e.g., 065 may be written 65, 607 ≠ 67).	P I U H	K2 K6	1a 1e			
The student knows that the value of a numeral is the sum of the products of each digit and its place value (expanded notation).	I U H	K3 K7	1a 1e			
The student knows that numbers written in any base system can be expressed in expanded notation as a sum of products.	I U H	K2 K7	1a 1b			
The student is able to rename a numeral in any base using expanded notation (e.g., $234_{(5)} = s \times 10^{2}_{(5)} + 3 \times 10^{1}_{(5)} + 4 \times 10^{0}_{(5)}$).	I U H	P35	1a 1b	5a		
The student is able to count in any base (i.e., order numerals in a base).	I U H	P34	1a 1d			(C) Numeration
The student is able to compute (add, subtract, multiply, and divide) in non-ten bases.	I U H	P35	1b 1e	5a		
The student knows an algorithm to change the base in which a number is written.	I U H	K2 K7	1b 1e			

2. Numeration

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
2.1 Base Systems (Cont.)						
The student is able to express a numeral written in one base as a numeral written in another base (e.g., $14_{(5)} = 1001_{(2)}$).	I U H	P35 P44	1b	5a		

2. Numeration

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
2.2 Decimal Systems						
The student knows that the term decimal system means that the system has base ten (grouping by tens).	P I U H	K1 K2	1a 1e		1.0	
The student knows that a numeral specified without a base indicator is usually assumed to be a base ten numeral.	I U H	K2	1a 1e		1.0	
The student knows the word names associated with place value in a decimal numeral (e.g., ones, tens, tenths, hundreds, hundredths).	P I U H	K1 K2	1a		1.0	
The student knows the Hindu-Arabic word names for the base ten counting numerals are zero, one, two, three, four, five, six, seven, eight, nine.	P I	K1 K2	1a		1.0	(C) Numeration
The student is able to express a decimal numeral both in words and with Hindu-Arabic numerals (e.g., 123 or one hundred twenty-three; 1.23 or one and twenty-three hundredths).	P I U H	P35 P41	1a 1b 1e	5a	1.0	
The student knows that decimal numerals are grouped with commas, in sets of threes, from right to left to represent the cyclic place value characteristics of larger numerals (e.g., 214,625,728).	P I U H	K3 K6	1a 1e 2c		1.0	
The student is able to separate a decimal numeral into groups of thousands to facilitate reading the numeral (i.e., can use commas as grouping symbols).	P I U	P33 P41	1a 1e 2c		1.0	
The student knows that the place value of any position in a decimal numeral can be expressed as a multiple (power) of ten (e.g., in 234, the 2 is in the third position which has value 10^2).	P I U H	K3 K7	1a 1b 1e		1.0	

2. Numeration

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
2.2 Decimal System (Cont.)						
The student knows that the number denoted by any digit in a decimal numeral is the product of the digit and its place value (e.g., in 624, the 6 has the value of 6×10^2 or 600).	P I U H	K3 K7	1a 1b 1e		1.0	
The student is able to rename a number either in decimal form or in expanded notation (e.g., $34.5 = 30 + 4 + \frac{5}{10}$ or $(3 \times 10) + 4 + (5 \times \frac{1}{10})$ or $(3 \times 10) + (4 \times 1) + (5 \times \frac{1}{10})$ or $(3 \times 10^1) + (4 \times 10^0) + 5 \times 10^{-1}$	P I U H	P35	1b	5a		
The student knows the rules for rounding off a number.	P I U H	K2 K7 K9	1e 2c 4a 7a 9a		1.0	
The student is able to round off a number to a given number of units in a given place (4046 to the nearest ten is 4050).	P I U H	P35 P36	1e 2c 4a 7a 9a	5a	1.0	

2. Numeration

COURSE GOALS	Level F/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
2.3 Other Systems						
The student knows the value associated with each symbol used in the Roman numeral system: I, V, X, L, C, D, M.	P I U	K2	1a 1c 1e 3			
The student knows that the order of the symbols in a Roman numeral indicates addition or subtraction.	P I U	K2	1a 1d 1e 3			(C) Sequence
The student is able to count in the Roman numeral system.	P I	P34	1a 1e 3	5a		
The student is able to express a number using Hindu-Arabic or Roman numerals.	P I U	P35 P66	1a 1e 3	5a		
The student knows the characteristics of a finite system (such as the integers 1 through 12 on the clock).	P I U H	K3	1a 9b		6.3	(C) Finiteness math

2. Numeration

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/(V1, V2) Value Words
2.4 Scientific and Exponential Notation						
The student knows that exponential notation is an expression of the form m^n (m to the nth power), where m is referred to as the base and n as the exponent.	I U H	K1 K2	1a		1.8 4.4	
The student knows that any real number may be used as an exponent.	U H	K2 K3 K7	1a 1b			
The student knows the definition of b^n (including when n is a natural number, zero, a negative integer or a fraction).	I U H	K2 K6	1a 1b 1c		1.8 4.4	
The student knows the meaning of the terms "squared" and "cubed."	I U H	K1 K2	1a		1.8	
The student knows the Laws of Exponents used in simplification and computation.	U H	G1 K7	1a 1b 1d 1e		1.4 1.8 4.4 4.5	
The student is able to perform operations with exponential expressions written in the same base.	U H	P35 P44	1a 1b 1d 1e	5a	1.4 1.8 4.4 4.5	
The student knows that raising a number to the nth power, where n is a positive integer, and finding the nth root are inverse operations (e.g., squaring and finding the square root are inverses).	U H	K3 K5	1a 1d		1.4 1.8 4.4 4.5	
The student knows that expressing a number in scientific (standard) notation means naming a number as the product of a decimal fraction with one digit to the left of the decimal point and a power of ten (e.g., 6.38×10^2).	U H	K1 K2 K7	1a 1b		4.5 4.6	
The student is able to name a number either in decimal form or scientific (standard) notation (e.g., 63.8, 6.38×10^1).	U H	P35 P44 P66	1a 1b	5a	4.5 4.6	

2. Numeration

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concern/ (V1, V2) Value Words
2.4 Scientific and Exponential Notation (Cont.)						
The student is able to use scientific notation and the Laws of Exponents to estimate the solution of a quantitative problem.	U H	P35 P36	1b 1c 4a	5a	4.5 4.6	
The student is able to rename a numeral in expanded notation using exponents ($6.27 = 6 \times 10^0 + 2 \times 10^{-1} + 7 \times 10^{-2}$).	I U H	P35	1a 1b	5a	1.0	

2. Numeration

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/(V1, V2) Value Words
2.5 Rational Number Names						
The student knows that a rational number may be expressed in many forms (i.e., $1\frac{1}{2}$, 150%, $\frac{3}{2}$, $\frac{6}{4}$, 1.5, $1 + \frac{1}{2}$).	I U H	K2 K5	1a 1b		1.3	(C) Equivalence
The student knows the definition of the key terms associated with rational numbers, including proper and improper fractions, mixed number, equivalent fraction, decimal fraction, percent, ratio, and proportion.	I U H	K1 K2	1a		1.3	
The student knows a method for converting a numeral from one of the forms, fraction, decimal, or percent, to an equivalent numeral in one of the remaining forms.	I U H	K2 K7	1b 1c 1e		1.3	(C) Equivalence
The student is able to rename a rational number in one of the forms, fraction, decimal, or percent.	I U H	P35	1b 1c 1e	5a	1.3	
The student is able to order rational numbers written in different forms (i.e., fraction, decimal or percent, $\frac{2}{3} > .3$).	I U H	P34 P35	1a 1b 1c	5a	1.3	(C) Ordinality
The student knows that there is an infinite set of fractional numerals that name the same number)	I U H	K3 K5	1b		1.3	(C) Infinity, math (C) Equivalence
The student knows that equivalent fractions are produced by multiplying a fraction by different names for 1. (e.g., $\frac{2}{3} = \frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$, $\frac{4}{8} = \frac{4}{8} \times \frac{\frac{1}{4}}{\frac{1}{4}} = \frac{1}{2}$).	P I U H	K2 K7	1b 1c 1e		1.3	

2. Numeration

2.5 Rational Number Names (Cont.)

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
The student is able to rename a number in fractional form with a set of different fractional names (equivalent fractions).	P I U H	P35	1b 1c 1e	5a		(C) Equivalence
The student knows a method for converting from a mixed numeral to a fraction or from a fraction to a mixed numeral.	I U H	K2 K7	1a 1b 1c 1d 1e		1.3	
The student knows that fraction, decimal and percent are all word names for rational numbers.	P I U H	K1 K3	1a		1.3	

2. Numeration

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/(V1, V2) Value Words
2.6 Number Theory						
The student knows that a prime integer is an integer p, $p > 1$, whose only divisors are +1, -1, +p, and -p.	U H	K1 K2 K3	1a 3		1.0	
The student knows that a prime number is a whole number greater than one, whose whole number divisors are only itself and one.	P I U H	K1 K2	1a 3		1.0	
The student knows that the set of prime numbers is not finite (i.e., there is no largest prime).	P I U H	K3	1a		1.0	(C) Finiteness, math
The student knows that composite integers are integers greater than one that are not prime.	P I U H	K1 K2 K5	1a		1.0	
The student knows that to determine whether an integer is prime involves testing divisibility by all primes less than or equal to the square root of the integer.	I U H	K7 K9	1a 1d		1.0	
The student knows that any composite number may be expressed uniquely as a product of prime factors.	I U H	K3 K8	1a 1b 1d		1.0	
The student is able to rename a number in prime factored form.	I U H	P35	1b 1e	5a	1.0	
The student is able to express a composite number in prime factored form (e.g., $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$).	P I U H	P35 P44 P66	1b 1e	5a	1.0	
The student is able to use divisibility tests as an aid in factoring composite numbers.	P I U H	P35 P47	1a	5a	1.0	

2. Numeration

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
2.6 Number Theory (Cont.)						
The student knows that when a number is written as a product of integers, any of the integers is a divisor of that number.	P I U H	K3	1a		1.0	
The student knows that the greatest common divisor (GCD) of two or more numbers is the largest of the set of common divisors of the numbers (also called greatest common factor, GCF).	I U H	K1 K2	1a		1.0	
The student is able to use the greatest common divisor (GCD) to reduce fractions.	I U H	P35	1b 1c 1e		1.0	
The student knows that a multiple of an integer is any number that is divisible by that integer.	P I U H	K2	1a		1.0	
The student knows that the least common multiple (LCM) of two or more numbers is the smallest of the set of common multiples of the numbers.	I U H	K1 K2	1a		1.0	
The student knows a method to find the greatest common divisor (GCD) and least common multiple (LCM) of two or more numbers.	I U H	K7	1c 1e		1.0	
The student is able to use the least common multiple (LCM) as an aid in computing with fractions (i.e., least common denominator).	I U H	P35	1e		1.0	
The student knows that the relation of congruency ($\cong$) for two integers implies that their difference is divisible by some integer, that is, for a, b, m integers $a \cong b \pmod{m}$ means $a - b$ is divisible by m).	H	K1 K2	1a			

2. Numeration

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/(V1, V2) Value Words
2.6 Number Theory (Cont.)						
The student knows that the relation congruent (≅) is reflexive, symmetric, and transitive.	H	K3	1a			(C) Equivalence
The student is able to determine congruency for two integers and a modulus.	H	P44 P47	1a	5a		
The student is able to perform operations with congruences (including addition, subtraction, and multiplication).	H	P35	1e	5a		
The student is able to determine to which mathematical system a set of integers modulo n belongs (e.g., does the class satisfy the properties of an integral domain).	H	P33 P47	1d			
The student knows the divisibility tests for certain positive numbers (e.g., 2, 3, 4, 5, 6, 8, 9, 10, 7, 11)	I U H	K7 K9	1e		1.1	
The student is able to use divisibility tests as one way to check computation.	I U H	P35 P47	1e	5a	1.0	
The student knows that every whole number is either even (divisible by two) or odd.	P I	K1 K2 K5	1a		1.0	
The student knows that an even number is any integer that can be expressed as $2n$, and an odd number is any integer that can be expressed as $2n + 1$, where n is an integer.	U H	K1 K2	1a		1.0	
The student knows that even and odd numbers alternate in the set of whole numbers.	P I	K3	1a		1.0	

3. Mathematical Sentences and Their Solutions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
3. Mathematical Sentences and Their Solutions						
The student knows the location and use of print and nonprint materials related to mathematical (number) sentences and their solutions (e.g., card catalog: "Mathematics"; *Reader's Guide*: "Mathematics"; Area and Building Audio-visual Catalogs: "Mathematics."	U H	K6	4b			(C) Resources, math (V1) Inquiry
The student knows a definition of a mathematical (number) sentence.	P I U H	K2	1a 4b			
The student knows that a number sentence may represent an analysis of a problem situation (word problem).	P I U H	K3	1a 4b			
The student knows a definition of an open mathematical sentence.	P I U H	K2	1a 4b			
The student is able to replace the frame or variable in an open sentence with a numeral or numerals which makes the sentence true (e.g., $7 + n = 9$, $n = 2$).	P I	P35 P44 P63	1b 1c 1d			

3. Mathematical Sentences and Their Solutions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
3.1 Equalities						
The student knows the Fundamental Theorem of Algebra: Every polynomial of positive degree over the complex numbers has at least one prime linear factor over the complex numbers.	H	G1 K2	1b 1c 2a 2b		1.5	(C) Structure, math
The student knows the meaning of equivalent sets.	P I U	K2 K3	1a 1c		7.0	(C) Equivalence
The student knows that an equation (equality) is a number sentence that has an equal sign (=) and represents two names for the same number.	P I U H	K2 K3	1a 1c			(C) Equivalence
The student knows properties of equality (e.g., reflective, transitive, symmetric).	I U H	K3	1c			(C) Equivalence
The student knows that any element of the solution set of an equation is a number that makes the equation true.	P I U H	K2 K3	1c 1d 1e			(C) Equivalence (C) Truth, math
The student knows the ways in which equations are classified (e.g., by degree, by number of variables).	I U H	K3 K5 K9	1a 1b			
The student knows that the solutions of equations may be restricted to a particular set (e.g., rationals, integers).	U H	K3 K7	1a 4b			
The student knows that for any equation the following are true: (1) the two members are equivalent if the same number is added to or subtracted from both members of the original equation, (2) the two members are equivalent if both members of the original equation are multiplied by or divided by the same number.	U H	K3 K7	1c 1d 4b			

3. Mathematical Sentences and Their Solutions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
3.1 Equalities (Cont.)						
The student is able to solve first degree equations in one variable by applying one or more of the following techniques: (1) collecting like terms, (2) removing grouping symbols, (3) adding an appropriate term to both members of the equation (4) multiplying both members of the equation by an appropriate number.	U H	P33 P35 P43 P63	1d 1e 4b	5a		
The student is able to graph a linear equation by using one of the following: (1) slope and y-intercept, (2) both intercepts, (3) point and slope, (4) set of points.	U H	P33 P35 P44 P76	1b 2a 2b	5a		
The student is able to graph a quadratic function by any of the following: (1) set of points, (2) vertex and x-intercepts, (3) vertex, y-intercept, and one other point.	U H	P33 P35 P44 P76	1b 2a 2b	5a		
The student is able to solve quadratic equations by factoring, completing the square, formula, and graphing.	U H	P35 P43 P63	1b 1c 1d 1e 2b 4b	5a		
The student is able to solve exponential equations by raising to powers, taking roots, using logarithms.	H	P35 P44 P63	2b	5a		
The student is able to use logarithms to solve power equations.	H	P35 P44 P63	2b 4b	5a		
The student is able to determine if any of the roots computed for an equation are roots that satisfy the original equation.	H	P33 P43 P47	1e 2b 4b	5a		

3. Mathematical Sentences and Their Solutions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Headings	Taxonomy (C) Concept/(V1, V2) Value Words
3.1 Equalities (Cont.)						
The student is able to use synthetic substitution to compute approximations of roots of equations of degree n $(n > 1)$.	H	P35 P44 P63	2b	5a		
The student knows that if a polynomial function with real coefficients has $a + bi$ as a root (a and b real, and $b \neq 0$), then $a - bi$ is also a root.	H	K3	1e 2b		1.5	
The student knows that every polynomial equation of degree n $(n \geq 1)$ with complex coefficients has exactly n complex roots.	H	K3	1c 2b			
The student is able to solve polynomial equations over the set of complex numbers.	H	P35 P63	1b 1c 1d 1e 4b	5a	1.5	
The student is able to use several methods to find a polynomial equation, given its roots.	H	P11 P24 P34 P35 P44 P45 P63 P75	1b 1c 1d 2a 2b	5a		

3. Mathematical Sentences and Their Solutions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
3.2 Inequalities						
The student knows the symbols for inequality (e.g., $\neq$, $>$, $<$, $\not>$, $\not<$).	P I U H	K2	1a 1c			
The student knows that a statement of inequality is a number sentence.	I U H	K3	1c 1d			
The student knows the properties of inequalities.	I U H	K3	1c			(C) Inequality
The student is able to solve linear inequalities algebraically and graphically.	U H	P35	1c 2a 2b	5a		
The student knows the meaning of quadratic inequality.	U H	K2	1a 1c			
The student is able to solve a quadratic inequality algebraically and graphically.	U H	P33 P35 P44 P63	1c 1d 1e 2a	5a		
The student is able to solve an absolute value inequality algebraically and graphically.	H	P33 P35 P44 P63	1c 2a 2b	5a		

3. Mathematical Sentences and Their Solutions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
3.3 Systems						
The student knows that systems of equations may contain more than two unknowns (i.e., n equations in n unknowns).	U H	K3	1a 1c 1d 2b			(C) Structure, math
The student knows solving a system of two equations in two unknowns means finding the set of all ordered pairs of real numbers that satisfy both equations.	U H	K7	1a 1c 1d 4b			
The student knows the methods for solving two equations in two unknowns (i.e., addition-subtraction, substitution, comparison, determinants, and graphing).	U H	K2 K7	1c 2b			
The student is able to solve a system of equations.	U H	P33 P35 P44 P63	1c 1d 1e 4b	5a		
The student is able to solve a system of inequalities.	U H	P33 P35 P44 P63	1c 4b	5a		
A student is able to graph a system of equations in two or three unknowns.	H	P35 P44 P76	1a 1c 1d 1e 2a	5a		
The student knows that linear programming is the solution of systems of linear inequalities with linear constraints for a maximum or minimum outcome.	H	K2 K7	1a 1c 1d 4b			
The student knows that computers may be used to solve large systems of equations and inequalities.	H	K7	1c 4b			

3. Mathematical Sentences and Their Solutions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
3.4 Word Problems						
The student is able to translate words in a problem into mathematical symbols (e.g., "How many are left?" indicates subtraction).	P I U H	P31 P35 P66	4b			(C) Symbols
The student is able to distinguish the relevant information in solving a word problem.	I U H	P33 P45	4b			
The student is able to infer the type of numbers ("range") that could validly be the answer of a word problem (i.e., if the problem involves positive fractions, the answer could be a positive fraction or a whole number).	I U H	K2 K3	4b			
The student is able to translate a word problem into equivalent mathematical equations or inequalities.	I U H	P33 P35 P44	4b			
The student is able to use suitable pictorial, graphic, or symbolic representation to aid in the solution of word problems (e.g., flow charts, set notation, graphs, Venn Diagrams).	I U H	P63 P76	4b	5a		
The student is able to estimate the answers to a word problem as a means of checking an answer.	I U H	P36 P63	4a 4b	5a	6.1	
The student knows that there is usually more than one way to organize and solve mathematical word problems.	I U H	K7	4b			
The student is able to solve word problems involving more than one step or operation.	I U H	P35 P63	4b	5a		

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3. Mathematical Sentences and Their Solutions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Headings	Taxonomy (C) Concept/(V1, V2) Value Words
3.4 Word Problems (Cont.)						
The student is able to use common formulas for the solution of appropriate word problems (e.g., I = PRT, V = lwh, P = 2L + 2W).	I U H	P35 P63	4b			
The student is able to solve problems involving life situations, home, investment, and business.	I U H	P35 P63	4b	5a		
The student is able to apply trigonometry to solve problems in carpentry, navigation, surveying, electronics, and building construction.	U H	P35 P63	4b	4a 4d 5a		
The student knows that the methods and concepts of derivatives and integrals may be applied to physical problems involving a rate of change (e.g., change of distance, velocity, volume, pressure, population, economic growth).	H	K2 K7	1a 4b	5a		

4. Relations and Functions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
4. Relations and Functions						
The student knows the location and use of print and nonprint materials related to relations and functions in mathematics (e.g., Area and Building Audio-Visual Catalogs: "Mathematics, Function"; Periodical: Mathematics Magazine.	H	K6	2a 2b			(C) Resources, math (V1) Inquiry
The student knows that a relation is a set of ordered pairs.	U H	K2	2a			(C) Relation
The student knows that a function is a relation in which no two ordered pairs have the same first coordinate.	U H	K2 K3 K5	2a			(C) Relation
The student knows the meaning of words used in working with functions (e.g., domain, range, sets, inverses).	U H	K1 K2	2a 2b			
The student knows that a function may be represented in many ways (e.g., graph, mapping, set of ordered pairs, formula).	I U H	K2	2a			
The student is able to apply the vertical-line test to a graph to determine if the graph represents a function.	U H	P47	2a 2b	5a		
The student knows definitions of an even function and an odd function (i.e., even: $f(-x) = f(x)$; odd: $f(-x) = -f(x)$).	U H	K2	2a 2b			
The student knows that a function f is periodic if there is a number p (a period of f), $p \neq 0$, such that for every x in its domain, $x + p$ and $x - p$ are in its domain, and $f(x + p) = f(x)$.	U H	K2	2a			

4. Relations and Functions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
4.1 Mapping						
The student knows that a mapping is a well-defined correspondence between the elements of two sets.	U H	K2 K5 K7	2a			(C) Relation
The student knows that the points on the Cartesian plane map one-to-one onto the set of ordered pairs of real numbers.	U H	K2 K7	1a 2a			(C) Relation (C) Equivalence
The student knows that a mapping can be represented by a graph.	U H	K2 K7	2a		5.3	
The student is able to represent a mapping graphically.	U H	P35 P44 P76	2a	5a		
The student knows that angle and distance preserving mappings (rigid motions) include translations, rotations, and reflections.	U H	K5	2a			
The student is able to map two sets in ways that represent different relations.	U H	P33	2a			

4. Relations and Functions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
4.2 Inverse Relations and Functions						
The student knows that the inverse of a relation is the set of ordered pairs obtained by interchanging the two components of every ordered pair in the relation.	U H	K2 K3	2a			(C) Relation
A student is able to graph a relation and its inverse.	H	P35 P76	2a 2b	5a		

4. Relations and Functions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
4.3 Algebraic Functions						
The student knows that a polynomial function is a function defined by a polynomial in one variable of any degree.	U H	K2 K3	1b 2a		1.81	(C) Structure, math
The student knows a function f is linear if there are numbers m and b such that $f(x) = mx + b)$	U H	K2	1c 2a 2b			
The student knows a definition for a power function ($f(x) = x^a$, $x > 0$ and a: real).	H	K2	1c 2a 2b		1.81	
The student is able to represent a polynomial function graphically.	U H	P35 P44 P76	1c 2a 2b	5a	1.81	
The student knows that f is a quadratic function if there are real numbers, $a \neq 0$, b and c such that $f(x) = ax^2 + bx + c$.	U H	K2	1c 2a 2b		1.81	
The student knows that a rational function is a quotient of two polynomial functions.	U H	K2	1c 2a 2b		1.8	
The student is able to graph rational functions.	U H	P35 P76	1c 2a 2b	5a	5.3	

4. Relations and Functions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
4.4 Exponential Functions						
The student knows the exponential laws as they apply over the set of real numbers.	U H	K2 K7	2a 2b			(C) Structure, math
The student knows a definition for an exponential function ($f(x) = a^x$, x: real and $a > 0$).	H	K2	2a 2b			
The student is able to evaluate and graph exponential functions.	H	P35 P76	2a 2b	5a		
The student knows the value of the number e and its use in exponential functions.	H	K2 K3 K7	1a 2a			

4. Relations and Functions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/(V1, V2) Value Words
4.5 Logarithmic Functions						
The student knows the definition of logarithms.	H	K2	1a 2a			(C) Structure, math
The student knows the vocabulary used in working with logarithms (e.g., mantissa, characteristic, interpolation, base, exponent).	H	K1 K2	1a 2a			
The student knows that ten is the base for common logarithms and e is the base for natural logarithms.	H	K2	1a 2a 2b			
The student knows that a logarithmic function is the inverse of an exponential function.	H	K5	2a 2b			
The student knows the laws of logarithms.	H	K2 K7	1e 2a 2b			
The student knows that finding the antilogarithm of a number is the inverse operation of finding the logarithm of a number.	H	K3 K5	1e 2a 2b			
The student is able to use tables to utilize logarithms in calculations.	H	P35	1e 2a 2b	5a		
The student is able to approximate answers to arithmetic problems of multiplication, division, root extraction and raising to powers using logarithms.	H	P35 P36	1e 2a 2b	5a		
The student is able to use linear interpolation to obtain a closer approximation than from a table of mantissas.	H	P35 P42	1e 2a 2b	5a		
The student is able to change logarithms from one base to another.	H	P35 P44	1b 2a	5a		

4. Relations and Functions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/(V1, V2) Value Words
4.5 Logarithmic Functions (Cont.)						
The student is able to graph a logarithmic function.	H	P35 P43 P76	2a	5a		

4. Relations and Functions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/(V1, V2) Value Words
4.6 Trigonometric (circular) Functions						
The student knows definitions of the trigonometric functions sine, cosine, and tangent (including in terms of a right triangle or a point in the coordinate plane).	U H	K2	2a			(C) Cyclic structure
The student knows the basic units for angular measure including degree and radian.	U H	K2	1b			(C) Measurement
The student is able to express angular measure in either degrees or radians.	H	P31 P35	1a 1b 1e	5a		
The student is able to use a table of values of trigonometric functions to choose the correct approximation for a given function.	H	P33 P36 P42	2a	5a		
The student is able to interpolate when using tables of trigonometric values.	H	P35 P43	2a 2b	5a		
The student knows the terms associated with angles including standard position, initial side, terminal side, sign, and magnitude.	U H	K2	2a			
The student knows that all trigonometric functions of an angle may be determined using a point on the terminal side of the angle.	U H	K7	2a 2b			
The student is able to extend the domain of the trigonometric functions by use of the unit circle and wrapping function (including functions of negative angles and angles greater than a right angle).	H	P44	2a 2b	5a		

4. Relations and Functions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
4.6 Trigonometric (circular) Functions (Cont.)						
The student is able to use reduction formulas to simplify trigonometric statements (e.g., sin 110° = sin 70° and sin (-20°) = -sin 20°).	H	P35 P45	1a 1b 2a 2b	5a		
The student knows the sign relationships for the trigonometric functions in each of the four quadrants.	H	K3 K5	2a 2b			
The student knows that trigonometric functions are cyclical.	H	K3	2a 5c			(C) Cyclic structure
The student is able to determine the domain and range of the trigonometric functions.	H	P33 P35	2a 2b	5a		
The student knows the graphic characteristics of the trigonometric functions (including period, amplitude, and phase angle).	H	K3	1b 2a 5c			
The student is able to graph the trigonometric functions.	H	P35 P76	2a 5c	5a		
The student is able to graph the inverses of the trigonometric functions.	H	P35 P76	2a 5c	5a		
The student knows the relationships among the trigonometric functions (including reciprocals and cofunctions).	H	K5	2a 2b			
The student is able to express a trigonometric function in terms of the other trigonometric functions.	H	P35 P44	1b 2a 2b			
The student knows the conditions which determine the numbers of solutions for a triangle.	U H	K2 K5 K6	2a 2b 4b			

4. Relations and Functions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
4.6 Trigonometric (circular) Functions (Cont.)						
The student knows the formulas for the solution of triangles including the laws of sines and cosines.	H	K7	2a 4b			
The student is able to solve triangles.	H	P35 P63	2a 2b 4b	5a		
The student is able to use the fundamental trigonometric function identities to prove other identities including Pythagorean and reciprocal identities.	H	P35 P42 P43 P44 P63	1b 2a 2b	5a		
The student is able to develop the multiple-angle formulas.	H	P35 P44	1b 2b	5a		
The student is able to solve trigonometric equations.	H	P35 P63	1e 2b 4b	5a		
The student is able to use a graph of a trigonometric equation to estimate the roots of a given function.	H	P35 P36	1b 4a	5a		
The student is able to use compact notation to represent the infinite roots of a specified trigonometric equation including using only principal roots.	H	P46	1a 2a	5a		

4. Relations and Functions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/(V1, V2) Value Words
4.7 Probability Functions						
The student knows the definition of a probability function (probability density function).	H	K2	2a		9.4	(C) Probability
The student is able to use a probability function to describe an experiment with random outcomes.	H	P33 P66	2a 2b	5a	9.4	
The student is able to use the probability function to calculate the variance and standard deviation of a random variable.	H	P35 P63	2a 2b	5a	9.4	
The student knows the definition of a probability distribution function.	H	K2	2a 2b		9.4	
The student knows the definition of a joint probability function.	H	K2	2a 2b		9.4	
The student knows the definition of a binomial probability function (binomial density function).	H	K2	2a 2b		9.4	
The student is able to use a binomial probability function to describe random experiments.	H	P33 P66	2a 2b	5a	9.4	
The student knows the definition of a Bernoulli probability function.	H	K2	2a 2b		9.4	
The student is able to use the Bernoulli probability function to describe coin tossing experiments with win-loss outcomes.	H	P33	2a 2b	5a	9.4	
The student knows that uniform probability functions, multinomial probability functions, hypergeometric probability functions, and Poisson probability functions are special probability functions.	H	K2 K5	2a 2b		9.4	

4. Relations and Functions

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
4.8 Other Functions						
The student knows the definition of the absolute value function.	H	K2	2a			
The student knows the definition of the greatest integer function.	H	K2	1a 2a			
The student is able to use the greatest integer function in proving limits of sequence.	H	P35	2a	5a		
The student knows the definition of a hyperbolic function.	H	K2	2a			

5. Geometry

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
5. Geometry						
The student knows the location and use of print and nonprint materials related to geometry (e.g., card catalog: "Mathematics," "Geometry"; Reader's Guide: "Geometry"; Area and Building Audio-Visual Catalogs: "Mathematics, Geometry."	H	K6	5a 5b 5c 5d 5e			(C) Resources, math (V1) Inquiry
The student knows that many mathematical systems including geometrical systems may be described in the following two ways: (1) a mathematical system consists of a) undefined terms, b) defined terms, c) postulates, and d) theorems, or (2) a mathematical system consists of a) a set of elements, b) one or more well defined operations, c) an equivalence relation, and d) a set of postulates.	I U H	G2 K2 K3	4b		1.0	(C) Structure, math (C) Mathematic systems
The student knows that geometry is a logically structured model of physical space dealing with size and shapes.	I U H	K2 K3	5d 5e			(C) Physical space (C) Model
The student knows that in the development of a geometrical system certain terms are accepted without definition (e.g., point, line, plane).	U H	K1 K3	5e			(C) Mathematic systems
The student knows that all geometric figures may be considered as sets of points, as subsets of lines, or planes, or as the union of these sets.	U H	K2 K3	5d 5e 6c 6d			
The student knows that concepts of geometry can be described algebraically.	U H	K3 K7	5c		5.3	(C) Equivalenc
The student knows that axioms or basic postulates are fundamental statements accepted without proof.	U H	K2	6c 6d			

5. Geometry

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
5. Geometry (Cont.)						
The student knows the basic postulates which establish the structure of a particular mathematical system.	U H	K2	5d 6b 6c 6d			(C) Structure, math (C) Mathematical systems
The student knows the prerequisite for mathematical reasoning is a set of precisely stated assumptions (basic postulates) from which conclusions (theorems) can be derived.	U H	K7 K8	5d 6b 6c 6d		8.4	(C) Structure, math
The student knows that assertions about geometric figures must be based on mathematically logical conclusions stemming from already accepted fact (basic postulates and theorems).	U H	K7	5d 6b 6c 6d		8.4	(C) Structure, geometric
The student knows mathematically valid deductions are dependent on the basic postulates and will change as the postulates change.	U H	K5 K8	6a 6b 6d			(C) Structure, math
The student knows that it is desirable for mathematical definitions to have the following properties: (1) name the term that is being defined, (2) use only terms that have been previously defined unless it is an undefined term, (3) place the term that is being defined into the smallest or nearest set to which it belongs, (4) state the characteristic of the defined term which distinguishes it from all other members of the set to which it belongs, (5) state the least number of essential properties required to identify the term being defined, and (6) be reversible, that is, when the subject and predicate nominative are interchanged, the resulting statement be true.	I U H	K2 K3	6b		8.4	

5. Geometry

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/(V1, V2) Value Words
5. Geometry (Cont.)						
The student knows that only definitions, postulates, and proven theorems can be cited to support assertions made in geometric proofs using deductive reasoning.	U H	K3 K9	6a 6b 6c 6d			(C) Proof
The student knows that a lemma is a proven conclusion which is helpful in the proof of a longer, more complicated theorem.	U H	K1 K2 K7	6c 6d 6e		1.0 5.5	
The student is able to use axiomatic-deductive reasoning in the discovery or proof of geometric facts.	U H	P42 P46 P52 P53	5d 6a 6c 6d		8.0	(C) Reasoning
The student knows that induction is the process of finding a general principle based upon the evidence of specific cases.	U H	G2 K2	5d 6d		1.0 8.0	(C) Reasoning
The student is able to develop a mathematical argument in the indirect or direct method of proof.	U H	P33 P43 P53 P62	5d 5e 6c 6d	5a	8.4	

5. Geometry

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
5.1 Plane and Space						
The student knows the theorems and postulates that establish a point, a line, and a plane.	U H	K3 K9	5b 5e 6c			(C) Physical space
The student knows the plane separation postulate.	U H	G1 K3	5b			(C) Relationship plane
The student knows the theorems, definitions and terms related to line segments (e.g., end points, betweenness, bisector, intersection).	U H	G1 K2 K7	5d 5e 6c 6d			
The student knows the definitions for term dealing with angles (e.g., angle, acute, obtuse, right, supplementary, complementary, interior, exterior, measure of an angle, linear, pair, vertical, dihedral).	U H	K2 K3	5e			
The student knows the postulates and theorems dealing with an angle (e.g., An angle has exactly one bisector; every angle corresponds to a unique real number such that $0 < x < 180$).	U H	G2 K3	5e 6d			
The student knows the conditions that must be established to demonstrate angles congruent (e.g., same measure, complements of congruent angles).	U H	K3	5e 6c			(C) Congruence
The student is able to establish the relationship between angles.	I U H	P33 P34 P45	5b 5e	5a		(C) Relationship angles
The student is able to use geometric terms (line, point, segment, ray, angle, vertex) in defining geometric figures.	I U H	P31 P33	5e	5a		

5. Geometry

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
5.1 Plane and Space (Cont.)						
The student knows that the nonempty intersection of two geometric figures forms a third geometric figure (e.g., 2 lines in a plane intersect to form a point, 2 planes in space intersect to form a line).	I U H	K8	5e 6c			(C) Relationshi geometric figures
The student is able to classify by name geometric figures in a plane and in 3-space (e.g., triangle, polygon, polyhedron, pyramid, sphere).	P I U H	P31 P33 P45	5e	5a		
The student knows the names of the parts of commonly studied geometric figures (e.g., diagonal, altitude, great circle).	P I U H	K1 K3	5a 5e			
The student knows that a geometric figure divides a plane or 3-space in distinct sets of points.	I U H	K5 K7 K8	5a 5b 5e	5a		(C) Physical space
The student knows that triangles are classified by their side and angle measurements (e.g., equilateral, isosceles, scalene, right, acute, obtuse, equiangular).	P I U H	K5	5e			(C) Measurement
The student knows the Pythagorean Theorem.	I U H	G1	5e 6c	5a	1.5 1.7 4.6	
The student is able to apply the Pythagorean Theorem in the solution of problems.	I U H	P35 P63	5a 5e	5a	4.6	
The student knows the formulas for finding the perimeters and areas of polygons	U H	K2	5a 5e	5a	6.0	
The student is able to find the perimeter and area of polygons.	P I U H	P35 P63	5a 5e	5a	6.0	(C) Dimensionality

5. Geometry

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/(V1, V2) Value Words
5.1 Plane and Space (Cont.)						
The student is able to find surface area and volume of geometric space figures.	U H	P35 P63	5a 6a	5a		
The student knows that two figures which are the same size and shape are congruent.	I U H	K2 K3	5e			
The student knows that congruency is an equivalence relation.	U H	K3	2a		4.1	(C) Equivalence-congruence
The student knows that corresponding parts of congruent geometric figures are congruent.	U H	K3	5e			(C) Congruence
The student knows the congruency theorems for proving triangles congruent (e.g., ASA, SSS)	U H	G1	5d 6c 6d			(C) Congruence
The student is able to use the congruency theorems to prove triangles congruent.	U H	P43 P47 P53	5e 6c 6d			(C) Congruence
The student is able to use congruent triangles in complex proof.	U H	P43 P53 P62 P63	5d 5e 6c 6d			
The student is able to use his knowledge of the triangular inequality theorem to solve problems involving triangles.	U H	P63	5d 5e	5a		(C) Relationship-space
The student knows the relationship between the angles opposite noncongruent sides and conversely.	U H	K3 K8	5d 5e 6c			
The student knows the theorems and properties related to the perpendicularity of lines and planes.	P I U H	G2	5d 5e 6c			

5. Geometry

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
5.1 Plane and Space (Cont.)						
The student knows the theorems that establish the parallel relationship between lines and/or planes (including alternate interior angles, corresponding angles).	U H	G2 K3	5d 5e 6a 6c		5.2	
The student knows the effect the parallel postulate has on the development of Euclidean geometry.	U H	K8	3 6c		5.3	
The student knows the Parallel Postulate.	U H	G1	5e			(C) Relationshi lines
The student is able to prove lines and/or planes parallel.	U H	P43 P53 P63	5d 5e 6c	5a		(C) Relationshi lines and plane
The student knows that the set of quadrilaterals contains parallelograms, rectangles, rhombuses, squares, and trapezoids.	P I U H	K1 K5	5e			
The student knows the conditions that must be established to demonstrate similarity between geometric figures (e.g., triangles are similar when they have 2 pairs of corresponding congruent angles).	U H	G2 K3	5e			
The student knows corresponding sides of similar geometric figures have the same ratio.	U H	K3	5e			
The student is able to determine when two geometric figures are similar.	P I U H	P43 P45	6c			(C) Similarity
The student knows the definition of terms regarding circles (including circle, secant, radius, diameter, chord, circumference, arc, tangent, tangent circles, concentric circles, minor arcs, major arcs, central angle, inscribed angle, radian, pi).	P I U H	K1 K2 K3	5e			

5. Geometry

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
5.1 Plane and Space (Cont.)						
The student knows the theorems regarding chords, secants, and tangents of circles.	U H	G1 K3	5d 5e 6c			
The student is able to find the measurement of the angles formed by intersecting chords, secants, and tangents of circles.	I U H	P35 P63	5e 9a 9b	5a		(C) Measurement
The student is able to use the theorems regarding circles in proofs and application problems.	U H	P43 P53 P63	5d 5e 6c 6d	5a		
The student knows that geometric construction is making a drawing using only a compass and a straight edge.	U H	K2	5b			
The student is able to use the basic constructions in making the incenter, circumcenter, orthocenter, centroid, and nine point circle.	U H	P76	4b 5b	5a		
The student is able to use the basic constructions to construct the medians and altitudes of a triangle, and a circle containing any three non-collinear points.	U H	P76	4b 5b	5a		
The student knows that the terms point, line, plane, and space represent abstract concepts.	P I U H	K1 K3	5b 5e 6c			(C) Physical space
The student knows definitions of commonly used terms that are related to the study of the point, line, plane, and space (e.g., ray, end point, half line).	I U H	K1 K2	5e 6d			
The student knows that not all geometric figures may be constructed.	I U H	K3	4b 5e			

5. Geometry

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
5.2 Coordinate Geometry						
The student knows the relationship between equations and their geometric representations in coordinate geometry.	U H	K3	5c			(C) Relations
The student knows the appropriate terminology associated with the study of the rectangular coordinate plane.	U H	K1 K2	5c			
The student knows that a plane rectangular coordinate system is the one-to-one correspondence between the set of all points in a plane and the set of all ordered pairs of real numbers (coordinate plane).	U H	K2 K3	5c 5e			(C) Coordinate system (C) Relationshi equivalence
The student knows the term locus as a set of points representing the solution set of a set of geometric conditions.	U H	K3	5c			
The student knows the geometric representation of the locus of an equation is called the graph of the equation.	U H	K1 K2	5b 5c 5e			
The student is able to test for the various kinds of symmetry possible in the construction of graphs or evaluation of a locus of points.	H	P43 P47 P62	5b 5c 5e	5a		(C) Symmetry
The student knows the standard form for the equation of a line in a plane and in space.	U H	K3 K7	5c			
The student is able to derive the slope of a line from the following conditions: (1) points on the line, (2) a line to other lines, (3) relationship between coefficients of the variables of the equations for the line.	H	P35	5b 5c 5e	5a		

5. Geometry

5.2 Coordinate Geometry (Cont.)

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
The student is able to develop the various forms for writing the equation of a line knowing only the coordinates of two points of the line.	U H	P35	5c			
The student is able to determine perpendicularity or parallelism between two or more lines.	U H	P35 P63	5b 5e	5a		
The student knows that a family of lines is a collection of lines with a common geometric property.	U H	K3	5c			
The student is able to evaluate the condition which makes a set of lines a family.	U H	P43 P47	5b	5a		
The student knows the distance formula(s) for computing the distance between 2 points in a plane; in 3-space	U H	P35 P42 P52	5d 5e			
The student knows that given any two points P and Q of a line, there is a coordinate system for the line such that the coordinate of P is zero and the coordinate of Q is positive.	U H	K3	5a 5b		1.0	
The student knows that corresponding to any two points A and B there is a unique positive number called their distance.	U H	K3	5b			(C) Measurement
The student is able to determine distance in the plane or 3-space.	U H	P35 P63	5b 5e	5a		
The student is able to find the point which divides a line segment into a desired ratio (point of division formula).	U H	P35 P63	5b 5e	5a		

5. Geometry

5.2 Coordinate Geometry (Cont.)

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
The student is able to prove many of the theorems of Euclidean geometry by analytic (algebraic) methods.	H	P42 P43 P53 P63	5c 5d			(C) Proof
The student knows that a conic can be obtained by cutting a right circular cone by a plane.	H	K8	5b			
The student knows the terminology related to conics (e.g., cone, intersection, circle, hyperbola).	H	K1 K2	5c			
The student is able to graph conic sections on the coordinate plane.	H	P33 P35 P44 P76	5b 5c	5a		
The student is able to convert the general form of a quadratic equation to the standard form of a specific conic equation.	H	P33 P35	5c	5a		
The student is able to express the conics, as well as the line, as parametric equations of a specific locus.	H	P33 P35 P44	5c	5a		(C) Equivalence
The student is able to find the points of intersection of two or more plane geometric figures algebraically and graphically.	U H	P35 P44 P63 P76	5b 5c 5e	5a	3.3	
The student knows the polar representation for points in a plane.	H	K2	5c		1.7 4.6	
The student is able to convert rectangular coordinates to polar coordinates and conversely.	H	P35 P44	5c	5a	1.7 4.6	
The student is able to graph equations using a polar coordinate system.	H	P35 P66	5b	5a		

5. Geometry

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
5.2 Coordinate Geometry (Cont.)						
The student is able to represent graphically both the original and the image of a conic which has been translated and/or rotated to make it identifiable.	H	P32 P66	5b 5c	5a		
The student is able to apply translations and rotations to a quadratic equation written in general form in order to identify the conic represented by it.	H	P33 P35 P44	5c	5a		
The student knows that the rectangular coordinate system can be extended to a system in three dimensions.	H	K3 K4	5c			
The student is able to sketch the graphs of space figures, including their traces in the three mutually perpendicular planes.	H	P33 P35 P76	5c	5a		

5. Geometry

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
5.3 Transformational Geometry						
The student knows that a transformation is an operation such that each point in the preimage set has an unique image and each point in the image set is the image of exactly one point.	H	K2 K3	5e			(C) Operations (C) Transformation
The student knows that three types of transformations are reflection, translation, and rotation.	H	K1 K5	5e			(C) Transformation
The student knows the relationship between a point and the reflection of the point with a line.	H	K3 K5 K8	5e			(C) Relationshi line-point
The student knows that the reflection, translation, and rotation transformations preserve angle measure, collinearity, and distance.	H	K3	5e 6c			(C) Equivalence
The student knows that if a figure coincides with its image over a reflecting line, then the figure is called reflection-symmetric to that line.	H	K2	5e			(C) Symmetry
The student knows that parallelity and perpendicularity are preserved when reflected over a line.	H	K3	5e			
The student knows that a composite of transformations result from successive applications of transformations.	H	K3 K8	5e 6c			
The student knows that a motion which preserves the properties of a figure is called an isometry.	H	K2 K3	5e			
The student is able to use isometries to prove the congruency of triangles.	H	P43 P53 P63	6c 6d			(C) Congruence
The student is able to use transformation postulates, definitions, and theorems to prove the theorems of plane geometry.	H	P43 P53 P63	6c 6d			(C) Proof

5. Geometry

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/(V1, V2) Value Words
5.4 Other Geometries						
The student knows that two people, Bolyai and Lobachevski, were basically responsible for the independent development of non-Euclidean geometry.	H	K1 K6	3			
The student knows that by changing Euclid's Parallel Postulate the non-Euclidean geometries may be developed.	H	G1 K7 K8	3 5d			(C) Structure of geometry
The student knows that Riemannian geometry contradicts Euclid's Parallel Postulate in that it assumes there are no parallel lines.	H	K3 K8	3 5d			(C) Structure of geometry
The student knows that Riemannian geometry is useful in applied mathematics and physics (e.g., the mathematical basis for Einstein's General Theory of Relativity).	H	K7	3 5d			
The student knows that in "neutral geometry" neither Euclid's Parallel Postulate is assumed nor its contradiction; all the rest of Euclid's postulates are assumed.	H	K3	3 5d			(C) Structure of geometry
The student knows that in the study of Lobachevskian geometry the Postulates of Euclidean geometry are assumed, but Euclid's Parallel Postulate is replaced by Lobachevski's Parallel Postulate: "If point P is not on line L there are at least two lines through P which are parallel to L."	H	G1 K3	3 5d			
The student knows the properties unique for projective geometry.	H	K3	5d			

6. Measurement

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
6. Measurement						
The student knows the location and use of print and nonprint materials related to measurement in mathematics (e.g., card catalog: "Mensuration," "Weights and Measures;" Subject Guide to Books in Print: "Measure Theory;" Reader's Guide: "Measurement;" Area and building audiovisual catalogs: "Measurement, Mathematics," "Measurement System, Metric," "Measurement, Instrumentation."	P I U H	K6	9a 9b			(C) Resources, mathematica (V1) Inquiry
The student knows that measurement is a comparison between a standard unit and the object to be measured.	P I U H	K2	9b			(C) Measurement
The student knows the term measure means the number of times a unit is used to determine a measurement.	P I U H	K2	9b			(C) Measurement
The student knows that measurement is a number associated with an abstract property such as length.	P I U H	K2 K8	9b			(C) Measurement
The student is able to express a measurement with a number.	P I U	P44	9b	5a		
The student knows common historical measurements from which present systems of measurement evolved (e.g., cubit, span, hand, rod).	P I U H	K4	9b		10.0	(C) Measurement
The student knows that the system most widely used in the United States today is the British-American system of measures.	I U H	K1 K6	3 9b			
The student knows the functions of the Bureau of Standards (including calibration of measurement devices and maintenance of standard units).	I U H	K7	9a 9b			

6. Measurement

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
6. Measurement (Cont.)						
The student knows that measurement may be implied through the use of words such as long, thin, or tall.	P I	K1	9b			(C) Measurement
The student is able to describe measurements in comparative terms such as long, thin, or tall.	P I	P45	9b			
The student knows that historically mathematics began with the need to measure and to record measurements.	I U H	K4 K8	9b		10.0 5.0	(C) Measurement
The student knows that environmental factors may affect measurement properties of an object (e.g., temperature and pressure may affect volume, location on the earth or moon affects weight).	I U H	K6 K8	9a			
The student knows that measurement may be non-quantitative (e.g., location by longitude and latitude; proportionality by scale drawings).	I U H	K3	9b			
The student knows that defined relationships may exist between different categories of measurement (e.g., rate is a function of time and distance).	I U H	K8	9a 9b			(C) Environment

6. Measurement

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Headings	Taxonomy (C) Concept/ (V1, V2) Value Words
6.1 Systems and Categories						
The student knows that different countries of the world use different measurement systems.	P I U H	K5	9a 9b			(C) Measurement
The student knows the names of the major systems of measurement (e.g., metric, British-American).	I U	K1	9b			
The student knows that number operations are required to convert measurements from one system to another.	I U H	K7	9a			(C) Measurement
The student knows that the metric system is a decimal system (based on units of ten).	I U H	K3	9b			
The student knows the names of the principal categories to which measurement is applied (including length, area, volume, weight, time, money).	P I U H	K1 K5	9b			(C) Measurement
The student knows that some categories of measurement have units common to many systems (e.g., unit for time such as hour).	I U H	K5	9a			(C) Equivalence
The student knows that a standard unit is the basis for deriving other units in a category.	I U H	K9	9a			(C) Measurement
The student knows that standard units of measure are arbitrarily chosen (i.e., agreed upon by a large number of people).	I U H	K2	9a			(C) Measurement
The student knows the standard unit for each of the major categories of the principal systems of measurement (e.g., dollar in the American monetary and linear meter in the metric system).	I U H	K2 K9	9a 9b			

6. Measurement

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
6.1 Systems and Categories (Cont.)						
The student knows the names of, and the actual physical dimensions suggested by, units of measure within the principal systems of measurement (e.g., inch, hour, meter, degree, pint).	P I U H	K1 K2	5a 9a 9b			
The student knows the names of the common units in the principal categories of measurement (e.g., money is measured with pennies, nickels, dimes; weight is measured in ounces, pounds, tons).	P I U	K1	9b			
The student knows accepted abbreviations and symbols for units of measurement (e.g., lb. or # for pound, $ for dollar).	P I U H	K2	9b			
The student knows the equivalent values of units within a category of measurement (e.g., 1 foot = 12 inches).	P I U H	K8	9a			
The student knows that equivalent units of measure are in the same dimension (i.e., inches cannot be converted to pounds or dollars, but only to feet or yards).	I U H	K2	9a			(C) Measurement (C) Equivalence
The student is able to convert a measurement in a particular unit to other units within a system (e.g., 4 yards = 12 feet).	I U H	P35 P44	9a	5a		
The student is able to perform basic operations (+, -, x, ÷) with measures (e.g., 3 hours, 45 minutes plus 2 hours, 20 minutes equal 6 hours, 5 minutes).	I U H	P35	9a	5a		
The student is able to measure using formulas (e.g., volume = V= L x W x H).	I U H	P35 P63	9a	5a		

6. Measurement

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
6.1 Systems and Categories (Cont.)						
The student knows the equivalent values of related units in different systems (e.g., 1 meter is about 3.3 feet).	I U H	K2	9a			
The student is able to convert measurements of the same dimension from one system to another (e.g., meters in the metric system to feet in the American system).	U H	P35	9a 9b	5a		

6. Measurement

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
6.2 Accuracy						
The student knows the terms associated with accuracy of measurement (including tolerance, precision, relative error, and greatest possible error).	I U H	K1	9a			
The student knows that measuring units may be microscopically small to astronomically large.	P I U H	K3	9b			(C) Measurement
The student knows that the smaller the measuring unit used the more precise is the measurement.	P I U H	G2	9b			(C) Measurement
The student knows that an estimated measure is an approximation which is rounded off to the nearest unit applicable.	P I U H	K2	9b			
The student knows that all measurements involve a possible error of half the value of the smallest division of the scale used in measuring.	I U H	K2 K3	9b			(C) Measurement
The student knows that the relative error of a measurement is the quotient of the greatest possible error and the measurement itself.	U H	K2 K7	9b			
The student knows the rules for computation with approximate numbers.	U H	K2 K7	9b			

6. Measurement

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
6.3 Measurement Instruments						
The student knows that approximate instruments are needed for measuring (e.g., a ruler to measure length, a clock to measure time, a thermometer to measure temperature).	P I U H	K7	4c 9a			(C) Measurement (C) Instrumentation
The student knows factors that affect the precision of a measuring instrument (e.g., scale division, physical construction, readability).	I U H	K8	4c 9a			
The student knows that all measures are approximate (i.e., there is always a difference between the actual measurement and that which is indicated by the measuring instrument).	P I U H	G2	9a			
The student is able to interpolate a recording between units on the scale of a measuring instrument.	I U H	P42	1e 9b	5a		
The student is able to find a measurement using an appropriate physical device (e.g., rule, watch, thermometer, scale, electric meter, speedometer).	P I U H	P42 P47	9a 9b	5a		

7. Sets

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
7.0 Sets The student knows the location and use of print and nonprint materials related to sets in mathematics (e.g., card catalog: "Aggregates" (mathematics); Reader's Guide: "Mathematics"; Area and building audiovisual catalogs: "Set, Mathematical," "Set, Algebraic."	P I U H	K6	2a 2b			(C) Resources, mathematica (V1) Inquiry

7. Sets

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
7.1 Terminology						
The student knows that a set is a well-defined collection of objects.	P I U H	K2	1a			(C) Classification
The student knows the symbols used in set notation (including ε, $\notin$, $\subset$, $\cap$, $\neq$, $\emptyset$, $\{\ \}$, $\cup$).	P I U H	K2	1a			
The student knows the meaning of terms used in describing sets (including complement, null, subset, universal set, infinite set, finite set, superset, equivalent sets, disjoint sets).	P I U H	K1 K2	1a			
The student knows ways in which the members of a set may be specified, including listing the members (elements) and describing the members by rule.	P I U H	K7	1a			(C) Classification
The student is able to use the symbolism of sets (set builder notation) to name a set (e.g., $\{ x : \geq 2 \text{ and } X \in R\}$).	U H	P33 P35 P46	1a	5a		
The student knows that the cardinal number of a set A, denoted n(A), is the number of elements the set contains.	P I U H	K2	1a		1.15	(C) Cardinality
The student knows that a one-to-one correspondence between two sets is a pairing which assigns to each member of each set one and only one member of the other.	P I U H	K2 K7	1a			(C) Relations
The student knows that in counting the number of members of a set, a one-to-one correspondence is made between the members and a subset of the counting numbers.	P I	K7	1a			(C) Numeration
The student is able to establish a one-to-one correspondence between two infinite sets.	I U H	P33 P44	1a			(C) Infinity

7. Sets

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Headings	Taxonomy (C) Concept/(V1, V2) Value Words
7.2 Operations on Sets						
The student knows definitions of the set operations including union, intersection, complementation, and Cartesian product.	I U H	K2	1a 1d			
The student knows the laws of operations on sets, including the commutative associative, distributive, identity, and complement laws.	U H	K2 G2	1a 1d			
The student is able to perform basic operations on sets including union, intersection, and Cartesian products.	P I U H	P33 P35	1a 1d 4b 6c 7c	5a	9.1 8.4	
The student knows ways to describe set operations including Euler circles, mappings, and Venn diagrams.	I U H	K7	1a 1d		9.1 8.4	
The student is able to relate sets graphically (e.g., union with Venn diagrams, subset with Euler circles).	I U H	P33 P66	1a	5a		
The student knows the parallelism that exists between set operations and operations in other areas (e.g., logic, circuit design, information theory).	H	K5	1a 4b 5c			
The student knows areas of mathematics in which set theory and symbolism are applicable (e.g., probability, logic, and geometry).	U H	K5 K6	1a 4b 5c			

8. Logic

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
8. Logic The student knows the location and use of print and nonprint materials related to logic in mathematics (e.g., card catalog: "Logic, Symbolic and Mathematical"; Reader's Guide: "Logic, Symbolic and Mathematical"; Area and building audiovisual catalogs: "Mathematics, Logic."	H	K6	6a 6b 6c 6d 6e			(C) Resources, mathematica (V1) Inquiry

8. Logic

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
8.1 Terminology						
The student knows that a simple logical proposition (statement) is a sentence that is either true or false, but not both.	I U H	K2	6a			(C) Logic (C) Truth, mathematica (C) Reasoning
The student knows the meaning of the terms used to describe logical propositions including axiom, theorem, lemma, and corollary.	U H	K1 K2	6a			
The student knows that the logical connections are "and," "or," and "not."	U H	K2	6a			
The student knows that a proposition applied to elements connected by the logical connection "and" is posited of both elements.	U H	K2	6a			
The student knows that the expression "one or the other or both" is called the "inclusive or."	U H	K1 K2	6a 6c			
The student knows that the expression "one or the other but not both" is called the "exclusive or."	U H	K1 K2	6a 6c			
The student knows that the negation of a logical statement is formed by preceding the statement with the logical connective "not."	I U H	K2 K7	6a 6b 6c			
The student knows the symbols for negation, conjunction, disjunction, conditional and biconditional.	U H	K2	6a			
The student knows that the logical operation of disjunction of statements in logic is analogous to the operation of union in set theory.	U H	K2 K5	6a 6c		7.2	
The student knows that the logical operation of conjunction of statements in logic is analogous to the operation of intersection in set theory.	U H	K2 K5	6a 6c		7.2	

8. Logic

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
8.1 Terminology (Cont.)						
The student knows that in logic the operation of negation is analogous to finding the complement of a set.	U H	K2 K5	6a 6c		7.2	
The student knows the truth tables for the connectives (negation, conjunction, disjunction), conditional and biconditional.	H	K2	6a			
The student knows the meaning of quantifiers including "all" and "some."	I U H	K2	6a			

8. Logic

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
8.2 Statements						
The student knows that an open sentence is a statement which contains a variable and becomes a proposition when an element is substituted for the variable.	H	K1 K2 K7	6a			
The student knows that compound statements are simple statements combined with logical connections.	U H	K2 K3	6a			
The student knows that a compound statement formed with the logical connective "and" is a conjunction.	U H	K2	6a 6d			
The student knows that a compound statement formed with the "inclusive or" is a disjunction.	U H	K2	6a 6d			
The student knows that logical statements of the form, "if p then q" are conditional statements (i.e., implications).	U H	K2	6a 6c			
The student is able to use "sufficient," "necessary," "if," and "only if" language in expressing conditionals.	U H	P33 P75	6a	5a		
The student knows that logical statements of the form, "p if and only if q" are biconditionals.	U H	K2	6a 6c			
The student knows that the conditional "if p then q" has a converse, "if q then p."	U H	K2	6a			
The student knows that the conditional, "if p then q" has the inverse, "if not p, then not q."	U H	K2	6a			
The student knows that the conditional "if p then q" has the contrapositive "if not q, then not p."	U H	K2	6a			

8. Logic

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
8.2 Statements (Cont.)						
The student is able to use truth tables to test the truth or falsity of compound statements.	U H	P45 P47	6a 6b 6c 6d		8.4	(C) Truth, mathematica
The student knows that statements that have the same truth tables are called logically equivalent statements (i.e., tautologies).	U H	K2	6a			(C) Equivalenc

8. Logic

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/(V1, V2) Value Words
8.3 Reasoning						
The student knows the mathematical use of the words "and," "or," and "not."	I U H	K2	5d 6a		5.0 7.2	(C) Reasoning
The student is able to form the converse, inverse, and contrapositive of a given implication.	U H	P35	6a 6d	5a		
The student is able to use Venn diagrams to clarify logical arguments.	I U H	P47 P76	6a 6d	5a	7.2	
The student is able to use indirect reasoning in logical arguments (i.e., a proof of the contrapositive of a theorem is a proof of the theorem).	U H	P53	6a 6c 6d			
The student knows that a proposition containing a finite number of variables and operations is a tautology if and only if it is true for every substitution of its variables.	H	K2 K7	6a 6c 6d			
The student knows that at any step of a logical argument a statement may be replaced by a logically equivalent one.	U H	K7	5d 6a 6c 6d		5.0	
The student is able to use the transitivity property of the conditional (syllogism) in logical arguments (i.e., if p implies q and q implies r, then p implies r).	U H	P44	5d 6a 6c 6d		5.0	
The student is able to use the rule of detachment (modus ponens) in logical arguments (i.e., if p is true and p implies q, then q is true).	U H	P44 P47 P62	5d 6a 6c 6d		5.0	
The student is able to use "modus tollens" in logical arguments (i.e., if not q is true, and p implies q then not p is true).	U H	P47	5d 6a 6c 6d		5.0	

8. Logic

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
8.3 Reasoning (Cont.)						
The student is able to use proof by contradiction (reductio ad absurdum) in logical arguments.	U H	P45	5d 6a 6c 6d		5.0	(C) Logic (C) Reasoning
The student is able to use logical arguments in geometric and algebraic proofs.	U H	P51 P53	5d 6a 6c 6d		5.0	(C) Logic (C) Reasoning
The student knows that Boolean Algebra is an abstract mathematical system that is useful in solving logical problems.	H	K2 K3 K6	1a 1d 6a 6c 6d		8.4 7.0	(C) Logic (C) Reasoning
The student is able to use Boolean Algebra to solve logical problems.	H	P45 P47	1a 1d 6a 6c 6d	5a	8.4 7.0	(C) Logic (C) Reasoning
The student knows common mathematical paradoxes (e.g., Zeno's paradoxes).	H	K3	6b			(C) Logic (C) Reasoning

8. Logic

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
8.4 Abstract Mathematical Systems						
The student knows the definition of an abstract mathematical system.	H	K2	1a 1d			(C) System, math (C) Structure, math
The student knows the definition of an equivalence relation.	H	K2	1c		1.0	(C) Equivalence
The student is able to use the concepts of an abstract mathematical system in studying other possible mathematical systems.	H	P33 P45	1a 1d 6a 6c			(C) System, math (C) Structure, math
The student is able to verify the field properties for subsets of the real numbers.	U H	P45 P47	1a 1d		1.4	(C) Structure, math
The student knows the definition and properties of a group.	H	K2 K3	1a 1b 1c			(C) Structure, math
The student is able to develop abstract mathematical systems that are groups.	H	P67	1b 1c 1d			(C) Structure, math
The student knows the definition of a ring.	H	K2	1a			(C) Structure, math
The student is able to develop abstract mathematical systems that are rings.	H	P67	1a			(C) Structure, math
The student knows the definition and properties of an integral domain.	H	K2 K3	1a 1c			(C) Structure, math
The student is able to determine whether a congruence class of integers (modular number system) is an integral domain.	H	P45 P47	1a			
The student knows the similarities and differences among groups, rings, integral domains, and fields.	H	K5	1b 1c 1d			(C) Structure, math

8. Logic

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
8.4 Abstract Mathematical Systems (Cont.)						
The student knows that a vector space may be defined as an abstract mathematical system of scalar values in operation with vector values.	H	K2	1a 1c			(C) System, mat
The student knows the definition and properties of Boolean Algebra.	H	K2 K3	1c 1e		7.0	(C) Logic

9. Probability and Statistics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
9. Probability and Statistics						
The student knows the location and use of print and nonprint materials related to probability and statistics in mathematics (e.g., card catalog: "Probabilities," "Statistics"; *Reader's Guide*: "Probabilities," "Statistics"; area and building audiovisual catalogs: "Mathematics, Probability," "Mathematics, Statistical."	I U H	K6	7a 7b 7c 7d 7e			(C) Resources, mathematica (V1) Inquiry
The student knows that among several events equally likely to occur, the probability that a given event will occur is the ratio of outcomes that produce the given event to all possible outcomes (e.g., the probability of rolling a 2 on a die is 1/6).	I U H	K2 K3	7a 7c			(C) Probability
The student knows that statistics is a body of methods for the systematic handling of data aimed at making decisions amidst uncertainty due to too much or too little data.	U H	K2 K3	7a 7b			(C) Data
The student knows that statistics enable probability limits to be placed on the truth of statements made about a total system or population from analysis of a sample.	U H	K10	7a 7c			(C) Probability (C) Statistics
The student knows that in order to avoid distortion of the results, statistical data must be treated in accordance with all statistical rules and formulas appropriate to the specific data.	H	K8	7b 7d			(V1) Honesty (V1) Integrity
The student knows that descriptive statistics consist of the collection, classification, and summarization of data into graphical displays and computed statistics about samples or whole populations for the purpose of drawing inferences about the population.	H	K2 K3	7a 7c			

9. Probability and Statistics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
9. Probability and Statistics (Cont.)						
The student knows definitions of terms associated with statistical inference (including reliability, validity, correlation, regression, covariance, distribution, level of confidence, level of significance, significance ratios or tests, types I and II errors, power of tests, standard error).	H	K1 K2	7a 7c			

9. Probability and Statistics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
9.1 Sampling and Data Collection						
The student is able to choose a sample representative of a population.	U H	P33	7a	5a		
The student is able to generate and use random numbers (e.g., tables).	H	P32	7a	5a		
The student knows that random sampling is only one means of obtaining a representative sample.	H	K5 K7	7a			
The student is able to choose random samples from a population.	U H	P33	7a	5a		
The student is able to construct line segment graphs to represent numerical data.	I U H	P46 P76	7a 7b	5a	9.4	
The student is able to construct a frequency table from numerical data.	I U H	P46 P76	7a 7b	5a		
The student is able to construct charts and bar graphs to represent numerical data.	P I U H	P46 P76	7a 7b	5a	9.4	
The student is able to construct a histogram and a frequency polygon for representing statistical data.	U H	P33 P34 P46 P76	7a 7b	5a	9.4	
The student knows the characteristics of the normal distribution.	U H	K2	7a 7b 7c		9.4	

9. Probability and Statistics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
9.1 Sampling and Data Collection (Cont)						
The student is able to determine the line of "best fit" for a set of random ordered pairs.	H	P76	7a	5a	9.4	
The student is able to use a line of best fit in estimating correlations and regression equations.	U H	P36 P37 P42	7a	5a	9.4	

9. Probability and Statistics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
9.2 Measures						
The student knows the common measures of central tendency of data such as mean, median, mode, and percentile.	I U H	K2	7a 7b			
The student is able to calculate the mean, median, and mode of numerical data.	I U H	P34 P35 P63	7a 7b	5a		
The student knows the common measures of variability of data such as range, standard deviation, and variance.	U H	K2	7a 7b			
The student knows the common measures of covariability of data such as the covariance and correlation coefficients.	U H	K2 K5	7a 7b			
The student is able to calculate the range, standard deviation, and variance from numerical data.	H	P34 P35 P63	7a 7b	5a		
The student is able to compute and interpret statistical measures of central tendency, variability, and covariability employing methods including confidence, level of significance, significance ratios or tests, Type I and Type II errors, power of tests, and standards.	H	P34 P35 P63	7a 7b	5a		
The student is able to calculate covariance and correlation coefficients.	H	P34 P35 P63	7a 7b	5a		

9. Probability and Statistics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
9.3 Combinatorial						
The student knows that permutations and combinations are useful for arranging data to compute the probabilities of events.	U H	K8 G2	7a			
The student knows that a permutation is a distinct arrangement of a set of elements.	U H	K2	7a			
The student knows that a combination is a set of elements considered without regard to the order in which the elements occur.	U H	K2	1a 7a 7c			
The student knows the fundamental principle of enumeration (i.e., multiplication principle of counting).	H	K2 G2	1a 7a			(C) Numeration
The student knows the meaning of symbols and terms associated with combinatorial mathematics (including factorial - !, $_nC_r$, $_nP_r$).	H	K1 K2	1a			
The student knows the formulae for computing numbers of permutations and combinations.	H	K7	1a			
The student is able to calculate the number of combinations or permutations for a stated condition.	U H	P35	1a 7a	5a		
The student is able to calculate special permutations (e.g., indistinguishable elements, circular permutations).	H	P35 P43	1a 7a	5a		
The student is able to use combinations in expressing the binomial theorem.	H	P35 P48	1a 1b 7a 7c	5a	1.81	

9. Probability and Statistics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
9.4 Probability Theory						
The student knows the definitions of the basic terms of probability (e.g., sample, sample point, sample space, event, experimental results, probability of an event).	U H	K2	7a			(C) Probability
The student knows that the outcomes of two probabilities are "equally likely" if they have equal chances of occurring.	I U H	K2	7a 7b			
The student knows that mathematical probability of an event may be represented as P(A) = cardinal number of A divided by the cardinal number of the sample space (e.g., P(A) = n(A)/n(S)).	H	K2 K7	7a			
The student is able to use tree diagrams in counting outcomes and computing probabilities.	I U H	P35	7c	5a		
The student knows the formula for the probability of the union of two events.	H	K2 K7	7c			
The student knows the probabilities associated with the union and intersection of events (e.g., P(E union F) = P(E) + P(F) - P(E intersection F)).	H	K2 K7	7c		7.2	
The student knows the probability of outcomes of ordinary uncertain events (e.g., tossing coin, rolling dice, drawing an inside straight).	I U	K2	7c			
The student knows the probabilities of outcomes with experiments using single solids of three or more faces.	I U	K2	7c			
The student knows the probabilities of outcomes with experiments using a pair of solids of three or more faces.	I U H	K2	7c			
The student knows the definition of odds (i.e., the odds in favor of A = probability of A divided by the probability of the complement of A).	U H	K2	7c			

9. Probability and Statistics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
9.4 Probability Theory (Cont.)						
The student knows the definition for the probability of the complement of an event (i.e., the probability of the complement of A is equal to 1 - P(A)).	H	K2 K7	7c			
The student knows that events are "mutually exclusive" if they have no points in common.	H	K2	7c			
The student knows that events are "independent" if and only if the probability of their intersection is equal to the product of their probabilities (i.e., the events have nothing to do with each other).	H	K2 K7	7c			
The student knows the definition of conditional probability (i.e., the probability of A given B is equal to the probability of the intersection of A and B divided by the probability of B, where $P(B) \neq 0$).	H	K2	7c			
The student knows the binomial theorem.	H	K2	1a 7c			
The student is able to use rank correlation to test the independence of two chance variables.	H	P35 P45 P47	1a 7c	5a		
The student knows a formula for calculating confidence intervals for probability functions of random samples.	H	K2 K7	7c		9.5	
The student knows Chebyshev's theorem.	H	K2 K7	7c			
The student knows the definition of Bernoulli trials.	H	K2	1a 7c			

9. Probability and Statistics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
9.5 Interpretation						
The student knows that the type of display chosen can affect the interpretation of statistical data.	I U H	K8	7a 7b 7c			
The student is able to interpret statistical information from a table or graph (including minimums or maximums and occurrence frequencies).	I U H	P45 P48	7b 7c	5a		

10. History of Mathematics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
10. History of Mathematics						
The student knows the location and use of print and nonprint materials related to the history of mathematics (e.g., card catalog: "Mathematics, History"; area and building audiovisual catalogs: "Mathematics, History."	I U H	K6	3			(C) Resources, mathematica (V1) Inquiry
The student knows ways in which mathematics throughout history has provided career opportunities (e.g., map making accounting, and surveying).	I U H	K6 K8	3	2a 3b 4b 4d 4f		(C) Relevance (V1) Usefulness
The student knows the different emphasis placed on mathematics by different cultures (e.g., Roman - practicality; Greek - imagination and fancy; Egyptian - religious; Arabian - trading).	P I U H	K8 G2	3	2a		(C) Culture (V1) Respect to view of others

10. History of Mathematics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
10.1 Early Period						
The student knows that counting was a basis for the development of arithmetic.	P I	K8	3		2.0	(C) Numeration
The student knows some of the early counting symbols (e.g., pictures, tally marks, groups of physical objects).	P I U H	K2	3		2.0	(C) Numeration
The student knows factors contributing to the development of numeration symbols (numerals) (including standardization of symbols and ease of record keeping).	I U H	K4 K8	3			(C) Numeration
The student knows the origin and development of the many common numeration systems (e.g., Roman, Greek, Hindu, Arabic, Egyptian (hieroglyphics), Chinese).	P I U H	K2 K4	3		2.0	(C) Numeration (C) Innovation (V1) Usefulness
The student knows that computation and number theory were primarily restricted to the scribes and priests of early civilization.	I U H	K6 G2	3			(C) Culture (C) Religion (V1) Intellectual Freedom
The student knows effects of beliefs about numbers upon religion, mysticism, and superstition (e.g., the divineness of odd numbers, the fear of 13, the mystical 666).	P I U H	K8	3			(C) Culture (C) Religion (V1) Objectivity
The student knows early mathematical games (e.g., odd and even, Nim, chess, draughts).	I U H	K2	3			(C) Education (C) Play (V1) Pleasure
The student knows reasons for the development of positional number systems (including the transition from simple counting to more complex computations).	I U H	K4 K8	3		2.1	(C) Numeration (C) Structure, math

10. History of Mathematics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
10.1 Early Period (Cont.)						
The student knows that man's first experience with mathematics was inductive (i.e., the Egyptians and Babylonians developed their ideas through observation and experimentation with concepts useful in their daily lives).	I U H	K2 K3	3		8.4	(C) Logic (V1) Objectivit
The student knows the ways that fingers (digits) have been used to compute and record numerical computations.	P I U	K2 K7	3			(C) Adaptation (V1) Innovativeness
The student knows factors contributing to the need for computation in certain areas (including the development of astronomy, commerce, surveying, science)	P I U	K4 K8	3	4b		(C) Adaptation (V1) Innovativeness
The student knows that man reasoned algebraically before he computed arithmetically.	I U H	K4 G2	3			(C) Logic
The student knows that Egyptian and Babylonian civilizations as early as 3000 B.C. were relatively advanced in arithmetic (e.g., simple algebra, numbers into the thousands, multiplication tables, weights and measures, calendars).	I U H	G2	3			(C) Culture (V1) Education
The student knows that the fractional concept developed when early civilized man began taking shared risks in trading and commerce.	I U H	K6 G2	3		2.5	(C) Relations (V1) Innovativeness
The student knows that early computation facts were recorded in tabular form.	P I U	G2	3			
The student knows that early civilized nations such as Egypt used geometric figures and properties for measuring (e.g., triangulation for surveying).	I U H	K7	3		5.1	(C) Adaptation (V1) Innovativeness

10. History of Mathematics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
10.1 Early Period (Cont.)						
The student knows the origin of some early computational devices (such as the abacus).	P I U H	K4	3		11.3	
The student knows the development of computing devices from grooves on walls, lines in sand and dirt, to the modern abacus and the electronic computer.	P I U H	K4	3		11.0	(C) Technology (V1) Innovativeness
The student knows that basic units of measure were developed from common objects found within a culture (e.g., cubic foot of water (talent) in Babylonia).	P I U H	K4 1 K8	3		6.0	(C) Culture (V1) Innovativeness
The student knows that geometry developed in Egypt because of the need to reestablish lost boundaries (surveying) washed out by the flooding Nile.	I U H	K7 K8	3		5.1	(C) Culture (C) Adaptation (V1) Justice

10. History of Mathematics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
10.2 Greco-Roman Period						
The student knows that the Greek mathematicians promoted the study of abstract mathematics contrasted to applied mathematics.	U H	K8	3			(C) Culture (V1) Sensitivit
The student knows that the main contribution of the Greeks was the process of ordered logic or deductive reasoning.	U H	K2	3		8.4	(C) Logic (V1) Human dignity
The student knows ways in which well-known Greek mathematicians contributed to civilization (e.g., Euclid, Archimedes, Pythagoras, Zeno, Eratosthenes, Thales, and Aristarchus).	I U H	K1 K8	3			(C) Culture (C) Civilizatio (V1) Innovation (V1) Human dignity
The student knows that the Greek disciples of Pythagoras formed a secret society (Pythagoreans) to promote the study of mathematics as a science.	I U H	K7 K8	3			(C) Learning (V1) Inquiry
The student knows that our present day geometry, Euclidian geometry, was developed by Euclid (323-285 B.C.) who collected, simplified, and recorded the geometric knowledge of his day.	U H	K8 G2	3		5.1 5.2	(C) Culture (V1) Innovativeness (V1) Usefulness
The student knows that Archimedes (297-212 B.C.) made great advances in applying mathematics to the analysis and solution of technical, engineering, and scientific problems.	U H	K1 K8	3			(C) Culture (C) Adaptation (V1) Innovativeness (V1) Usefulness
The student knows the contributions of Eratosthenes, a Greek mathematician (including measurement of the earth's size before 2000 B.C. and development methods for the study of prime numbers).	I U H	K1 K2	3		2.6	(C) Culture (V1) Innovativeness
The student knows that the major contribution of the Roman civilization was in practical applications such as engineering and accounting.	P I U H	K3 K8	3	2a 4b 4f		(C) Culture (V1) Usefulness

10. History of Mathematics

10.3 Middle Age Period

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
The student knows that the Arab culture and learning in mathematics spread to every major center of civilization during the 125 years following the death of Mohammed in 632 A.D.	I U H	K4 K6 G2	3			(C) Culture
The student knows ways mathematics was kept alive during the medieval ages by the Arabian civilization approximately 400-1000 A.D. (e.g., translating and preserving earlier mathematical works).	I U H	K4 K6	3			(C) Culture (V1) Respect fo cultural heritage
The student knows the origin and development of counting houses and the various non-pencil and paper devices used in mathematical computations (e.g., counting boards, wax slates, dust tables).	P I U H	K4	3			(C) Adaptation (V1) Usefulness
The student knows that modern notation is called Arabic notation but our notation involving zero was in fact developed by the Hindus of India.	P I U H	K1 K5 K8	3		2.0	(C) Numeration
The student knows ways in which the adoption of the Hindu-Arabic place value system simplified computation.	P I U H	K8	3		2.0	(C) Adaptation (C) Numeration (V1) Usefulness
The student knows how the decimal system of mathematics evolved.	P I U H	K2 K4	3		2.2	(C) Adaptation (V1) Usefulness
The student knows the origin and development of the common idea words in arithmetic (e.g., digit, compute, algorithm, abstract, composite, prime, ordinal, cardinal).	P I U H	K1 K2 K4	3		1.0	(C) Communicati (V1) Usefulness
The student knows the origin and development of the zero and place value concepts.	I U H	K2 K4	3		2.1	(C) Numeration (V1) Usefulness

10. History of Mathematics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
10.3 Middle Age Period (Cont.)						
The student knows the major effects of mathematics on Western culture (e.g., the contribution of mathematics and scientific technology to the creation of classes within a culture).	I U H	K8	3			(C) Culture

10. History of Mathematics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
10.4 Modern Period						
The student knows the ways in which the invention of the printing press affected the use of numbers and symbols in mathematics (e.g., standardizing numerals).	I U H	K2 K8	3			(C) Adaptation (V1) Usefulness
The student knows that mathematical ideas and systems may be developed that have no immediate practical application (e.g. the ellipse was known thousands of years before it was used by Kepler to predict the motions of planets).	P I U H	K4 K8	3			(C) Adaptation (V1) Beauty (V1) Sensitivity
The student knows that the metric system for measurement was developed in 1789 by the French Academy of Sciences.	I U H	K1 K4 K6	3		6.1	(C) Measurement (V1) Usefulness
The student knows that the modern period of mathematics began in the 1600's with the development of analytic geometry and calculus.	I U H	K4	3			(C) Culture
The student knows of the development of various mathematical branches (e.g., algebra, analytical geometry, calculus, and set theory).	U H	K4 K8	3			(C) Adaptation (V1) Usefulness
The student knows contributions of significant innovations in mathematics (e.g., complex numbers, logarithms, non-Euclidean geometries, Cartesian plane).	U H	K4 K8	3			(C) Adaptation (V1) Usefulness
The student knows the development of long division algorithm methods prior to their introduction in the 1800's. (e.g., Galby Method, Gerbent's Method, Scratch Method).	I U H	K2 K4	3			(C) Adaptation (V1) Usefulness

10. History of Mathematics

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
10.4 Modern Period (Cont.)						
The student knows that mathematical discoveries are sometimes made at the same time by different men working independently (e.g., Leibnitz and Newton on calculus).	I U H	K6	3			(C) Adaptation (V1) Creativity
The student knows that Karl Gauss, Isaac Newton, and Archimedes are considered by most math historians as the three greatest mathematicians of history.	I U H	K1 G2	3			
The student knows about the lives and contributions of several of the great mathematicians (e.g., Gauss, Descartes, Newton, Pascal).	I U H	G2	3			(C) Culture (V1) Respect fo cultural heritage
The student knows that Rene Descartes' mathematical ideas led to a new field of mathematics called analytic geometry, cited as one of the most useful ways scientists have of studying natural phenomena.	U H	K1 K8	3		5.3	(C) Adaptation (V1) Usefulness
The student knows that set theory was developed by George Cantor (1845-1918) as a new method of analysis and proof.	I U H	K1 K2 K8	3		7.0	(C) Adaptation (V1) Innovation
The student knows that group theory developed by Evariste Galois (c. 1830) applies to motions, numbers, or spaces.	U H	K1 K6	3			(C) Adaptation (V1) Innovativeness
The student knows important names and activities of mathematicians involved in the development of computer mathematics (e.g., Gottfried Von Leibnitz, Babbage, Lord Kelvin, Napier, Pascal, Von Neuman.	U H	K1 K4	3		11.1	(C) Adaptation (C) Culture (C) Technology (V1) Usefulness

11. Computational Devices

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
11. Computational Devices The student knows the location and use of print and nonprint materials related to computers in mathematics (e.g., card catalog: "Electronic Computers"; Reader's Guide: "Computers"; area and building audiovisual catalogs: "Computers."	I U H	K6	8			(C) Resources, mathematical (V1) Inquiry

II. Computational Devices

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
II.1 Computers						
The student knows the historical evolution of computers.	I U H	K4	3 8		10.0	(C) Culture (C) Adaptation (VI) Usefulness
The student knows characteristics of the two basic types of computers: analog and digital.	I U H	K3 K5	8			
The student knows the relative advantage of digital and analog computers.	U H	K5	8			(C) Technology (VI) Usefulness
The student knows the social and economic effects the computer has on our world (e.g., industrial automation, invasion of privacy, and automation of business processes).	U H	K8	3 8		10.0	(C) Technology (VI) Human dignity
The student knows that recent rapid changes in computer capabilities are largely due to the advances in technology (e.g., electronics).	U H	K4 K8	3 8	6a		(C) Technolog (VI) Progress
The student knows the opportunities and requirements that are associated with computer careers such as programmer, keypunch operator, computer operator, and systems analyst.	U H	K3 K6	8	3b 4a 4d 4f		(C) Economic system (VI) Human dignity (VI) Usefulnes
The student knows the meaning of basic terms associated with computers (e.g., hardware, software, central processing unit, peripheral devices, and memory).	I U H	K1 K2	8	4f		
The student knows types of computer hardware used for input, output, memory, and calculating.	I U H	K5 K7	8			
The student is able to use input-output devices to communicate with a computer system.	I U H	P35	8	5a		

11. Computational Devices

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
11.1 Computers (Cont.)						
The student is able to store data in and retrieve data from a complete storage system.	U H	P35 P66	8	5a		
The student knows characteristics of problems which are well-suited to computer solution.	I U H	K3 K6	8			
The student knows that the digital computer uses yes-no "decisions" based only on numerical computations.	U H	K7	8			
The student knows the general stages in computer problem solving (i.e., input, processing, and output).	I U H	K4	8			
The student knows that a digital computer can interpret only instructions written in machine readable form.	I U H	K2 K7	8			
The student knows the relationship between binary and octal numbers.	I U H	K5	8			
The student knows that a program is a series of instructions written in a language that can be interpreted and processed by a digital computer.	I U H	K2	8			
The student knows that programs written in a symbolic language are translated to machine usable form before computer execution.	U H	K2 K7	8			
The student knows terms associated with programming (including flow chart, decision table, statement types, languages, compilation, and documentation).	U H	K2	8			
The student knows the purposes of documenting a program (e.g., revision, testing, instruction).	U H	K7	8			

11. Computational Devices

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
11.1 Computers (Cont.)						
The student is able to flow chart a computer problem solution.	U H	P35 P66	8	5a		
The student is able to develop a program in a symbolic language from a flow charted solution.	U H	P35 P66	8	5a		
The student is able to use common programming procedures such as counters, branches, and loops.	U H	P35	8	5a		
The student is able to use a computer language to communicate with the computer (e.g., BASIC, FORTRAN).	U H	P66	8	5a		
The student is able to analyze errors within a program (debugging), (including printing results at various stages of processing and using system supplied diagnostics).	U H	P22 P43 P47	8	5a		
The student is able to use the language of arrays in programming (e.g., subscripting levels of data).	U H	P31 P35	8	5a		
The student is able to use appropriate instructions to arrange the physical display of printed output (format output).	U H	P35	8	5a		
The student knows the advantages of the various computer languages to specific applications (e.g., COBOL - Business, FORTRAN - Science).	U H	K5 K8	8			

11. Computational Devices

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
11.2 Slide Rule						
The student knows that the slide rule is an economical mechanical device used for real-number computations (other than addition and subtraction).	U H	K2 K6	1e 8	3b 4a		
The student knows that the slide rule is logarithmically arranged and therefore works according to the laws of exponents.	U H	K3 K7	8		4.5 2.4	
The student is able to read the slide rule scales.	U H	P41	8	5a		
The student is able to select the appropriate scales on the slide rule on which to perform particular computations.	U H	P45 P62	8			
The student knows the rules for computation with approximate numbers.	U H	K9	1e 8			
The student is able to use the slide rule to perform computations with real-numbers including multiplication, division, proportions, squaring, square root, cubing, and cube root.	U H	P35	1e 8	5a		
The student is able to use knowledge of scientific notation to place the decimal in the answer to a computation performed on the slide rule.	U H	P33 P35	1e 8	5a	2.4	

11. Computational Devices

COURSE GOALS	Level P/I/U/H	Knowledge or Process Classifications	Subject Area Program Goals	Career Education Program Goals	Other Related Content Taxonomy Headings	(C) Concept/ (V1, V2) Value Words
11.3 Other Devices						
The student knows that historically many devices have been used to facilitate computation (including Napier's bones, tables, and mechanical calculating machines).	P I U H	K4 K6	3		10.0	(C) Culture (C) Technology (V1) Usefulnes
The student is able to use devices to aid in calculations (e.g., abacuses, slide rules, nomographs, and tables).	P I U H	P35	8	3b 4a 4f 5a		
The student knows the ways in which calculators are classified (including by number of keys, function of keys, presence of memory (storage), and program capabilities).	I U H	K5	8			
The student is able to select a computing device appropriate to the task to be performed.	I U H	P33 P47	8	5a		
The student is able to use a desk calculator for the basic operations.	I U H	P35	8	4a 4d 4f 5a		
The student is able to add and subtract with an abacus.	P I	P35	1e 8		6.4	
The student is able to operate a programmable calculator.	H	P35	1e 8	4a 4d 4f 5a		
The student is able to program a programmable calculator (i.e., adapt the operations to particular problem analysis).	H	P35	8	4a 4b 4f 5a		